Asymmetric Information, Heterogeneity in Risk Perceptions and Insurance: An Explanation to a Puzzle

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Abstract

Competitive models of asymmetric information predict a positive relationship between coverage and risk. In contrast, most recent empirical studies find either negative or zero correlation. This paper, by introducing heterogeneity in risk perceptions into an asymmetric information competitive model, provides an explanation to this puzzle. If optimism discourages precautionary effort, there exist separating equilibria exhibiting the observed empirical patterns. It is also shown that zero correlation is consistent with information barriers to trade in insurance markets. The predictions of our model allow us to empirically distinguish it from standard asymmetric information models.

Key Words: Asymmetric Information, Insurance, Risk Perceptions

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Introduction

Most recent empirical studies of insurance markets have focused on the relationship between the coverage of the contract and the ex post risk (accident rate) of its buyers. The results are mixed. Fang, Keane and Silverman (2008) find strong negative correlation in the US Medigap insurance market. De Meza and Webb (2001) provide casual evidence for a negative relationship in the credit card insurance market. Cawley and Philipson (1999) study of life insurance contracts also shows a negative relationship which, however, is not statistically significant. They also report that insurance premiums fall with coverage even at the largest quantities. This finding suggests the absence of proportional loadings in the life insurance market. Chiappori and Salanie (2000) find a not statistically significant negative relation for the automobile insurance market. More recently, Finkelstein and McGarry (2006) analyze the long-term care insurance market and find no evidence of a positive relationship between insurance coverage and care utilization (actual risk). However, they find a positive relationship between perceived risk and coverage.

In contrast, in one of the earliest studies, Puelz and Snow (1994) find a positive relationship in the automobile insurance market in Georgia. Similar findings are reported by Finkelstein and Poterba (2002, 2004) for the UK annuities market. Individuals who purchase annuities tend to live longer than those who do not buy.

Starting with the seminal Rothschild-Stiglitz paper (1976), most theoretical models of competitive insurance markets under asymmetric information predict a positive relationship between coverage and the accident probability of the buyer of the contract. This prediction is shared by models of pure adverse selection (e.g. Rothschild and Stiglitz (1976)), pure moral hazard (e.g. Arnott and Stiglitz (1988)) as well as models of adverse selection plus moral hazard (e.g. Chassagnon and Chiappori (1997) and Chiappori et al. (2006)). In fact, Chiappori et al. (2006) show that the positive correlation property holds true in a very general framework involving multiple levels of losses, multidimensional adverse selection, moral hazard and even non-expected utility.

1 4.8% of U.K. credit cards are reported lost or stolen each year. The figure for insured cards is 2.7%.
2 In fact, they find that a fixed administrative cost and a constant marginal cost explain almost all risk-adjusted variation in prices (R-square values close to 1).
3 In the Chiappori and Salanie (2000) study those opting for less coverage purchase the legal minimum of third-party coverage. Dionne et al. (2001) look at contracts with two different levels of deductibles.
4 However, Dionne, Gourieroux and Vanasse (2001) argue that this result is likely to be a spurious effect of the linear specification in Puelz and Snow (1994).
However, de Meza and Webb (2001) provide a model where agents are heterogeneous with respect to their risk aversion and face a moral hazard problem. Also, insurance companies pay a fixed administrative cost per claim. In this model, there exist a separating and a partial pooling equilibrium predicting a negative relationship but due to the fixed per claim cost the less risk-averse agents go uninsured. The fact that the administrative costs are now incurred only by the insured agents changes the computation of the premiums which allows Chiappori et al. (2006) to derive their result.\textsuperscript{5}

Thus, in the presence of fixed administrative costs, competitive models of insurance markets under asymmetric information can potentially explain the observed negative or zero correlation between coverage and risk. However, their prediction cannot be consistent with negative or zero correlation in insurance markets where all agents opt for strictly positive coverage, there are just two events (loss/no loss), and no proportional loadings.\textsuperscript{6} Thus, the de Meza-Webb (2001) model cannot explain the empirical findings of Cawley and Philipson (1999).

Jullien, Salanie and Salanie (2007) provide an explanation for the negative correlation between coverage and risk that relies on market power rather than fixed costs. As far as the insurees are concerned, their model is similar to de Meza-Webb (2001) but in their case the insurer has monopoly power. In order to reveal their type and obtain insurance at a lower per unit price, the less risk-averse insurees accept low coverage. Moreover, not only are the more risk-averse agents willing to pay a higher per unit price to purchase more coverage but also take more precautions and so have a lower accident probability. The positive correlation property breaks because the insurer exploits his monopoly power and extracts more surplus from the more risk-averse insurees. However, more coverage is associated with a higher per unit price.

As a result, Jullien, Salanie and Salanie (2007) can explain the negative correlation between coverage and risk but the striking observation of Cawley and Phillipson (1999) that insurance premiums exhibit quantity discounts remains a puzzle.

This paper, by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information, provides an explanation to the puzzling empirical

\textsuperscript{5} Negative correlation equilibria may also arise in cases where there is more than one level of loss and one of the two contracts chosen in equilibrium offers more comprehensive coverage. Comprehensive insurance allows claims to be made for contingencies not covered by the less comprehensive contract (e.g. two different levels of deductibles). Therefore, the more comprehensive contract entails higher expected administrative costs that could result in the breaking of the positive correlation property.

\textsuperscript{6} See also Lemma 2 and the discussion in Section 3.
findings. Numerous empirical studies both by psychologists and economists indicate that the majority of people tend to be unrealistically optimistic, in the sense that overestimate their ability and the outcome of their actions and underestimate the probability of various risks. For example, Svenson (1981) finds that 90 percent of the automobile drivers in Sweden consider themselves “above average”. Similar results are reported by Rutter, Quine and Alberry (1998) for motorcyclists in Britain. On average, motorcyclists both perceive themselves to be less at risk than other motorcyclists and underestimate their absolute accident probability.

In several other studies people appear to overestimate their survival probabilities (Gan, Hurd and McFadden (2003)) and underestimate their susceptibility to health problems (Weinstein (1987)) and the health risk of smoking (Sutton (1998), Hammar and Johansson-Stenman (2004)). As these studies indicate, people do hold different beliefs about the same risk and, on average, they underestimate it. Furthermore, some authors present evidence indicating that optimism discourages precautionary effort. Finkelstein and McGarry (2006), Lee (1989), Lundborg and Lindgren (2002) and Viscusi (1990) find that those who perceive a higher risk tend to take more precautions.

In line with the empirical evidence, this paper drops the assumption that all insurees have an accurate estimate of their accident probability. It assumes that some agents, the optimists (henceforth Os), underestimate it both in absolute terms and relative to the less optimistic ones (henceforth Rs). They also tend to be less willing to take precautions. However, both the Os and the Rs are certain that their perceived probabilities coincide with the true ones and so their choices are determined by these (mistaken for the Os) beliefs. That is, insurees aim at maximising their expected utility given their beliefs.

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8 On the other hand, Viscusi (1990) finds that more individuals overestimate the risk of lung-cancer associated with smoking than underestimate it and, on average, they overestimate it. However, the overestimation of smoking risk reported by Viscusi (1990) may be due to the fact that he asked individuals to provide an estimate of the average smoker’s risk not their own risk. Weinstein (1998) reviews a sizable literature demonstrating that smokers perceive the risks to themselves from smoking to be substantially lower than the smoking risks faced by other smokers (see also Slovic (2000)).
9 Schulman et al. (1993) study twins and provide evidence that differences in the degree of optimism are intrinsic rather than an acquired characteristic.
10 See Landier and Thesmar (2003) for a discussion of theories developed to explain optimism.
11 Also, Coelho and de Meza (2005) provide experimental evidence that unrealistic optimism is prevalent and optimism and performance are negatively correlated.
12 A sizable literature has investigated the implications of overconfidence and unrealistic optimism in securities markets and firm financing. See Barberis and Thaler (2002) for a survey.
In this framework there exist separating equilibria exhibiting negative or zero correlation between coverage and risk. In the first case, the Os not only take fewer precautions (high-risk type) but also purchase less coverage than the Rs. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Because they underestimate their accident probability, the Os purchase low coverage at a high per unit price, although contracts offering more insurance at a lower per unit price are available. There also exists a separating equilibrium that exhibits zero correlation between coverage and risk and involves the Rs being quantity-constrained. In order to reveal their type, the Rs accept lower coverage than they would have chosen under full information about types.

These results have several interesting implications. First, they can explain both the negative correlation between coverage and risk and the fact that insurance premiums display quantity discounts reported by Cawley and Philipson (1999). This is a key distinguishing feature of our model. As we have argued above, neither the model by de Meza and Webb (2001) nor that by Jullien, Salanie and Salanie (2007) can simultaneously explain these empirical findings.

Second, according to Fang, Keane and Silverman (2008), heterogeneity in risk preferences cannot explain the strong negative correlation between coverage and risk in the US Medigap insurance market whereas heterogeneity in risk perceptions (overoptimism/overconfidence) can.

Third, they are consistent with the puzzling empirical findings of Finkelstein and McGarry (2006): the positive correlation between perceived risk and coverage and the simultaneous lack of a positive relation between coverage and actual risk.

Fourth, they suggest that a lack of a positive correlation between coverage and risk can be consistent with informational barriers to trade in insurance markets.

Finally, they provide us with a sufficient condition which can be used to empirically distinguish our approach from standard asymmetric information models even in the presence of proportional loadings. To this end, we rely on a revealed preference argument derived by Chiappori et al. (2006). Its validity is independent of the market structure, proportional loadings/taxes or whether some agents go uninsured.13 However, because some agents underestimate their risk, this prediction

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13 Given risk aversion and state-independent utility, this revealed preference argument may fail only if some agents underestimate their risk. For example, it holds true in the models of de Meza and Webb (2001) and Jullien, Salanie and Salanie (2007).
fails in both equilibria presented in this paper. Therefore, its rejection by the data is consistent with our model but not with asymmetric information models in which insurees accurately estimate their accident probability. That is, our model can be empirically distinguished from the latter models even if these models also predict negative or zero correlation between coverage and risk.\textsuperscript{14}

This paper is organised as follows. In Section 1, we present a model where agents differ with respect to their risk perceptions and face a moral hazard problem. Section 2 provides a diagrammatic proof for the existence of the two separating equilibria described above. Section 3 deals with the empirical implications of our results. Finally, section 4 concludes.

1. The Model

In order to illustrate the effects of optimism and to facilitate comparison with standard asymmetric information models, we employ a model similar to de Meza and Webb (2001).\textsuperscript{15} In particular, there are two states of nature: good and bad. In the good state there is no loss whereas in the bad state the individual (insuree) suffers a gross loss of $D$. Before the realisation of the state of nature all individuals have the same wealth level, $W$. Also, all individuals are risk averse and have the same utility function but differ with respect to their perception of the probability of suffering the loss. There are two types of individuals, the Rs and the Os. The Rs have an accurate estimate of their true loss probability whereas the Os underestimate it.\textsuperscript{16} However, both the Os and the Rs believe that they correctly estimate their accident probabilities and their choices are driven by these (mistaken for the Os) beliefs. That is, all agents maximise their expected utility given their beliefs.

Furthermore, all agents can affect the true loss probability by undertaking preventive activities. Given the level of precautionary effort, the true loss probability

\textsuperscript{14} The introduction of proportional loadings/taxes into competitive asymmetric information models could give rise to separating equilibria predicting negative or zero correlation between coverage and risk even if all agents buy some insurance. However, the revealed preference argument would still hold true. The empirical failure of this argument is a sufficient (but not necessary) condition for optimism.

\textsuperscript{15} Our model differs from de Meza and Webb (2001) in two respects: i) In our model, individuals differ with respect to their degree of optimism whereas in de Meza and Webb they differ with respect to their degree of risk aversion, ii) In our case, administrative costs are zero. In our model, the results are only driven by heterogeneous risk perceptions (optimism). All of our results hold true if instead of the accident probability the Os underestimate the loss size (overconfidence).

\textsuperscript{16} For expositional simplicity, we assume that the more optimistic agents are optimists whereas the less optimistic are realists. However, all the results go through if two types are respectively optimists and pessimists.
is the same for both types. We consider the case where agents either take precautions or not (two effort levels). If an individual takes precautions \((C_i = C)\), he incurs a utility cost of \(C\) and his true probability of avoiding the loss \(p(C_i)\) is \(p_c\). If he takes no precautions \((C_i = 0)\), his utility cost is 0 but his true probability of avoiding the loss \(p(C_i)\) is \(p_0\), where \(p_c > p_0\).

Now, let \(p_j^i\) \((i = O, R, j = C, 0)\) be the perceived probability of avoiding the loss. Given our assumption about the risk perceptions of the Os and the Rs, we have \(p_j^R = p_j\), \(p_j^O > p_j\) \((j = C, 0)\), where \(p_j\) is the true probability of avoiding the loss.

In this environment, the (perceived) expected utility of an agent \(i\) is given by:

\[
EU(C_i, y_i, \lambda_i, W) = p_j^i U(W - y) + (1 - p_j^i)U(W - D + (\lambda - 1)y) - C_i
\]

where \(W\): insuree’s initial wealth
\(D\): gross loss
\(y\): insurance premium

\((\lambda - 1)y\): net payout in the event of loss, \(\lambda > 1\)

\(\lambda y\): coverage (gross payout in the event of loss)

Hence, the increase in the (perceived) expected utility from taking precautions is:

\[
\Delta_i = \left(p_c^i - p_0^i\right)\left[U(W - y) - U(W - D + (\lambda - 1)y)\right] - C_i, \quad i = O, R
\]

where \(U\) is increasing and strictly concave and \(W - y\), \(W - D + (\lambda - 1)y\) are the wealth levels in the good and the bad state respectively.

Insurance companies know the true accident probability (given the precautionary effort level) and the perceived accident probabilities of the Os and Rs but they can observe neither the type nor the actions of each insuree. They also know the cost for the insuree corresponding to each precautionary effort level, the utility function of the insurees and the proportion of the Os and Rs in the population. The insurance contract \((y, \lambda y)\) specifies the premium \(y\) and the coverage \(\lambda y\). As a result, since insurance companies have an accurate estimate of the true accident probability, the expected profit of an insurer offering such a contract is:
\[ \pi_j = p_j y - (1 - p_j)(\lambda - 1)y, \quad j = C, 0 \quad (3) \]

**Equilibrium**

Here, we use the equilibrium concept employed by Rothschild and Stiglitz (1976). That is, we look for equilibria in a free-entry game with many insurers. A pair of contracts \( z_O = (y_O, \lambda_O y_O) \) and \( z_R = (y_R, \lambda_R y_R) \) is an equilibrium if the following conditions are satisfied: i) No contract in the equilibrium pair \( (z_O, z_R) \) makes negative expected profits. ii) No other set of contracts introduced alongside those already in the market would increase an insurer’s expected profits.

Depending on parameter values both separating and pooling equilibria can arise.\(^{17}\) Below, we provide a diagrammatical description of the two most interesting separating equilibria.\(^{18}\)

2. **Negative and Zero-Correlation Equilibria**

Let \( H = W - y \) and \( L = W - D + (\lambda - 1)y \) denote the income of an insuree who has chosen the contract \( (y, \lambda y) \) in the good and bad state respectively. Let also \( \bar{H} = W \) and \( \bar{L} = W - D \) denote the endowment of an insuree after the realisation of the state of nature.

2.1. **Effort Incentive Constraints**

Let us first consider the moral hazard problem an insuree of type i faces. A given contract \( (y, \lambda y) \) will induce an agent of type i to take precautions if

\[ (p'_C - p'_O)[U(H) - U(L)] \geq C \quad \Leftrightarrow \quad \Delta_i \geq 0, \quad i = O, R \quad (4) \]

Let \( P_i P'_i \) be the locus of combinations \( (L, H) \) such that \( \Delta_i = 0 \). Since \( C, \ U' > 0, \) the \( P_i P'_i \) locus lies entirely below the \( 45^0 \) line in the \( (L, H) \) space. This locus divides

\(^{17}\) For some parameter values the non-existence problem, known from Rothschild and Stiglitz (1976), also arises.

\(^{18}\) Details about the other equilibria are available from the author upon request.
the \((L, H)\) space into two regions: On and below the \(P_iP_i'\) locus the insurees take precautions (this is the set of effort incentive compatible contracts) and above it they do not. The slope of \(P_iP_i'\) in the \((L, H)\) space is given by:

\[
\frac{dL}{dH}|_{P_iP_i'} = \frac{U'(H)}{U'(L)} > 0 \quad \text{since} \quad U' > 0
\]

(5)

That is, \(P_iP_i'\) is upward-sloping. Also, since both types have the same utility function, the shape of \(P_iP_i'\) is independent of the type of the insuree.\(^{19}\)

However, the location of \(P_iP_i'\) does depend upon the insuree’s type. Although the Os overestimate their probability of avoiding the loss at any given precautionary effort level, they may either overestimate or underestimate the increase in that probability from choosing a higher preventive effort level. In principle, both cases are possible. However, if, given that no precautions are taken, the optimists’ perceived accident probability is very low, then the latter seems to be more reasonable. Also, this case is consistent with the empirical findings of Finkelstein and McGarry (2006), Lee (1989), Lundborg and Lindgren (2002) and Viscusi (1990). All these studies find that those who perceive a higher risk tend to take more precautions.\(^{20}\) In this paper, the analysis is carried out under the assumption that the latter case is relevant.\(^{21}\) In particular, the following assumption is made:

**Assumption 1:**

\[ p^R_c - p^R_0 > p^O_c - p^O_0 \]

That is, the Rs’ set of effort incentive compatible contracts is strictly greater than that of the Os. It should be noted that Assumption 1 is required for but does not necessarily imply a negative relationship between coverage and risk. It may well be the case that Assumption 1 holds and a separating equilibrium arises exhibiting a positive relationship. We also assume that

\(^{19}\)The curvature of these loci does not affect our analysis.

\(^{20}\)Coelho and de Meza (2005) also provide experimental evidence that optimism and performance are negatively correlated.

\(^{21}\)Details about the equilibria arising when optimism encourages precautionary effort are available from the author upon request.
Assumption 2: \[(p_C^i - p_o^i)\left[U(H) - U(L)\right] > C, \quad i = O, R\]

Assumption 2 implies that both $P_R^i P_R'$ and $P_O^i P_O'$ pass above the endowment point, and so the effective set of effort incentive compatible contracts is not empty for either type. If Assumption 2 does not hold for either type, the corresponding type never takes precautions. Although, Assumption 1 is necessary for the negative correlation prediction, Assumption 2 does not need to hold for the Os. In fact, this result obtains more easily if the Os never take precautions. In contrast, the zero-correlation result requires Assumption 2 but not Assumption 1.\(^{22}\) It obtains even if the Os overestimate the decrease in their accident probability from taking precautions.

2.2. Indifference Curves

The indifference curves, labelled $I_i$, are kinked where they cross the corresponding $P_i P_i'$ locus. Above $P_i P_i'$, insurees of the the i-type do not take precautions, their perceived probability of avoiding the loss is $p_o^i$, and so the slope of $I_i$ is:

\[
\left.\frac{dL}{dH}\right|_{t_i, p=p_o} = -\frac{p_o^i}{1 - p_o^i} \frac{U'(H)}{U'(L)} \quad i = O, R \tag{6}
\]

On and below $P_i P_i'$ insurees of the i-type do take precautions, their perceived probability of avoiding the loss rises to $p_C^i$ and so the slope of $I_i$ becomes:

\[
\left.\frac{dL}{dH}\right|_{t_i, p=p_c} = -\frac{p_C^i}{1 - p_C^i} \frac{U'(H)}{U'(L)} \quad i = O, R \tag{7}
\]

Hence, just above $P_i P_i'$ the i-type indifference curves become flatter.

Furthermore, because the Os underestimate their accident probability, at any given identical preventive effort level and $(L, H)$ pair, the Os indifference curve is steeper.

\(^{22}\) The zero-correlation result obtains even if the direction of the inequality in Assumption 2 is reversed. However, this assumption would imply that both types never take precautions and so this case is not very interesting.
in the \((L,H)\) space. Intuitively, the Os are less willing to exchange consumption in the good state for consumption in the bad state because their perceived probability of the bad state occurring is lower than that of the Rs.

2.3. Insurers’ Zero-profit Lines (Offer Curves)

Using the definitions \(H = W - y\) and \(L = W - D + (\lambda - 1)y\), and the fact that insurance companies have an accurate estimate of the true accident probabilities, given the precautionary effort level, the insurers’ expected profit function becomes:

\[
\pi_j = p_j(W - H) - (1 - p_j)(L - W + D)
\]  

(8)

The zero-profit lines are given by:

\[
L = \frac{1}{1 - p_j}W - \frac{p_j}{1 - p_j}H - D, \quad j = C,0
\]  

(9)

Conditional on the preventive effort level chosen by the two types of insurees, there are three zero-profit lines with slopes:

\[
\left. \frac{dL}{dH} \right|_{x=0} = -\frac{p_0}{1 - p_0} \quad \text{(EN’ line, effective from } N' \text{ to } P_oP_o')
\]  

(10)

\[
\left. \frac{dL}{dH} \right|_{x=0} = -\frac{p_c}{1 - p_c} \quad \text{(EJ’ line, effective from } E \text{ to } P_rP_r')
\]  

(11)

\[
\left. \frac{dL}{dH} \right|_{x=0} = -\frac{q}{1 - q} \quad \text{(EM’ line, effective between } P_rP_r' \text{ and } P_oP_o')
\]  

(12)

where \(q = \mu p_c + (1 - \mu)p_o\), and \(\mu\) is the proportion of the Rs in the population of insurees. Also, EM’ is the pooling zero-profit line when the Rs take precautions whereas the Os do not.

Also, at \(H = \overline{H} = W\), Eq. (9) becomes:
Eq. (13) is independent of the value of \( p_J \). This implies that all three zero-profit lines have the same origin (the endowment point, E).

We can now provide a diagrammatic proof of the existence of the two separating equilibria. The negative correlation result is shown in Proposition 1 whereas Proposition 2 provides an example of a separating equilibrium exhibiting zero correlation between coverage and risk.

**Proposition 1:** If the Os’ indifference curve tangent to \( E_N' \), \( I^*_O \), passes above the intersection of \( EJ' \) and \( P_R P_R' \), above the endowment point \( E \), and meets \( P_O P_O' \) above \( EJ' \), then there exists a unique separating equilibrium \( (z_R, z_O) \) where the Rs take precautions whereas the Os do not. Both types choose strictly positive coverage but the Rs buy more than the Os (see Fig. 1).²³

²³ This is true if i) the degree of optimism is sufficiently high so that the Os’ perceived probability of avoiding the loss is larger than the Rs’ true probability given that the Rs take precautions whereas the Os do not \( (p^O_0 > p^R_C) \), ii) the Os significantly underestimate the effect of precautions on the accident probability (the distance between \( P_R P_R' \) and \( P_O P_O' \) is large). iii) the difference between the true probabilities of avoiding the loss \( (p_C - p_0) \) is sufficiently small relative to the loss size.
**Proof:** Suppose that types are publicly observable but the effort level is not contractible. Then, given their perceptions about accident probabilities and the effect of precautionary effort on these probabilities, in a competitive equilibrium the Os take contract \( z_O \) and the Rs take contract \( z_R \). Also, notice that the Os strictly prefer \( z_O \) to \( z_R \) and the Rs strictly prefer \( z_R \) to \( z_O \). Thus, the revelation constraints of both types are satisfied (that is, both types choose the contracts they would have chosen under full information about types). Therefore, there is no profitable deviation and so \((z_R, z_O)\) is the unique equilibrium.\(^{24}\) Q.E.D.

Intuitively, given the contracts offered, because the Os considerably underestimate the reduction in their accident probability from taking precautions, they choose to take no precautions. Also, although insurance is offered at actuarially fair terms, because they underestimate their accident probability, the Os underinsure choosing a contract with low coverage while contracts with higher coverage are available at a lower per unit premium.

That is, this separating equilibrium exhibits two interesting features. The Os not only purchase less coverage than the Rs but also take fewer precautions and so their accident probability is higher. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Therefore, this equilibrium is consistent with both the negative correlation between coverage and risk (point estimate) and the fact that per unit premiums fall with the quantity of insurance purchased as reported by Cawley and Philipson (1999).

**Proposition 2:** Suppose \( EM' \) does not cut \( I_R \) through the intersection point of \( EJ' \) and \( I'_O \) (the Os’ indifference curve tangent to \( EJ' \) below \( P_O P'_O \) and to the left of \( E \)). Then there exists a unique separating equilibrium where both types purchase strictly positive coverage and take precautions but the Rs buy more insurance than the

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\(^{24}\) Here, we assume that the Os do not update their beliefs after observing the contract they are offered although the terms they are offered are, according to their beliefs, actuarially unfair. Alternatively, one could introduce an arbitrarily small fraction of agents whose true accident probability and effect of precautions on this probability coincide with those perceived by the Os. Now, the beliefs of the Os would not necessarily be shaken since they know that the terms they are offered are determined by the mass of optimists and they have a strong prior that they are one of the high ability agents (many empirical studies report that the overwhelming majority of people believe they are “above average” e.g. Svenson (1981)). In this case, in equilibrium, the contract taken by the Os would be arbitrarily close to \( z_O \) whereas the contract taken by the Rs would be exactly \( z_R \).
Os. That is, this equilibrium exhibits zero correlation between coverage and the accident probability (see Fig. 2).²⁵

Proof: Consider the following deviations. Clearly, offers above \( EJ' \) are loss-making. The same is true for offers above \( P_R P'_R \). Between \( P_R P'_R \) and \( P_O P'_O \) and below \( EJ' \) there is no offer that attracts Rs and does not attract the Os, although there are some offers that attract only the Os. Thus, any offer in this region is unprofitable. Given the equilibrium contracts, below \( EJ' \) and below \( P_O P'_O \) there is no offer that is attractive to either type. Hence, the pair \((z_R, z_O)\) is the unique separating equilibrium. Furthermore, the fact that \( EM' \) does not cut \( I_R \) below \( P_R P'_R \) rules out any pooling equilibrium. Therefore, the pair \((z_R, z_O)\) is the unique equilibrium. \( Q.E.D. \)

Although they buy more coverage than the Os, the Rs are quantity-constrained. Under full information about types, the Rs would have purchased the contract at the intersection of \( P_R P'_R \) and \( EJ' \), instead of \( Z_R \), which involves more insurance. However, since types are hidden, this contract is not offered because it violates the Os’ revelation and effort incentive constraints and so is loss-making for the insurance

![Fig. 2. Zero Correlation Equilibrium](image-url)

²⁵ This is true if i) the Os moderately underestimate the benefit of precautions (the distance between \( P_R P'_R \) and \( P_O P'_O \) is not very large) and ii) the degree of optimism is not very high (the Os perceived accident probability is not much smaller than the true one).
companies. In order to reveal their type, the Rs accept lower coverage than they would have chosen if types were observable.

Strictly speaking, the no-correlation prediction is unlikely to be observed in practice. However, if one interprets it as a failure to reject the no-correlation null, then it is consistent with the findings of Cawley and Philipson (1999), Chiappori and Salanie (2000) and Dionne et al. (2001) about the relationship between coverage and risk. Also, this equilibrium can simultaneously explain the positive correlation between perceived risk and coverage and the zero correlation between coverage and the accident rate (actual risk) reported by Finkelstein and McGarry (2006). Because the Os underestimate their risk they purchase less insurance than the Rs although, given the equilibrium contracts, both types have the same accident probability.

It should be pointed out that the results of both Proposition 1 and Proposition 2 hold true if we allow for fixed administrative costs (provided these costs are not very high). In this case, the equilibrium in Proposition 2 would predict zero correlation between coverage and risk and a negative relationship between coverage and per unit premiums. Since both types take precautions they have the same accident probability and so are charged the same marginal price. But the fact that the Os purchase less coverage implies that their total per unit premium is higher. This prediction fits very well with the finding of Cawley and Philipson (1999) that a fixed underwriting cost and a constant marginal cost explain almost all risk-adjusted variation in prices.

Finally, it is worth mentioning that both equilibria are inefficient. In the equilibrium of Proposition 1 the imposition of a minimum coverage requirement would lead to an increase in the Os’ (objective) welfare without affecting the Rs. Also, an intervention policy involving a fixed tax per contract sold, with the proceeds returned as a lump-sum subsidy to the whole population can yield a strict Pareto improvement on the laissez-faire equilibrium described in Proposition 2. Because the revelation constraint of the Os (high risks) is binding in Proposition 2, the welfare properties of the equilibrium and the intervention scheme used are similar to those in de Meza-Webb (2001). In contrast, in Proposition 1, because the Os are very optimistic no revelation constraint is binding and so the aim of the intervention is to make the Os buy more coverage as this does not create any negative externality on the

26 In the case of Proposition 1, objective probabilities are used as the welfare criterion whereas in Proposition 2 the result holds true regardless of whether objective or subjective probabilities are used. Details are available from the author upon request.
Rs. Thus, in this case, both the welfare properties of the equilibrium and the appropriate intervention scheme differ from those in de Meza-Webb (2001).

3. Implications for Empirical Testing

The results of Propositions 1 and 2 also allow us to empirically distinguish our approach from standard asymmetric information models. To this end, we employ a simplified version of the Chiappori et al. (2006) general framework. There are two states of nature: good and bad. In the good state the agent incurs no loss whereas in the bad state he incurs a loss of $D_\theta$. The parameter $\theta$ represents all the characteristics of the agent (potential insuree) that are his private information (risk, risk aversion, loss, etc). Agents may also privately choose their loss probability $1 - p$. This choice implies a prevention cost that is assumed to be a negative function of the loss probability. A contract consists of coverage and premium: $z = (\lambda y, y)$, $\lambda > 1$. The ex post risk of an insuree is a function of the contract he chooses. The average ex post risk of insurees choosing contract $z$ is $1 - p(z)$. In this general setup, the following assumptions are made:

A1: For all contracts offered and all agent types overinsurance is ruled out by assuming $\lambda y \leq D_\theta$.

A2: Agents’ preferences over final wealth are state-independent.

A3: Agents are risk averse in the sense that they are averse to mean-preserving spreads on wealth.

A4: Given the contract chosen, agents accurately estimate their accident probability.

A5: Insurance companies are risk neutral, and incur a fixed cost per contract $c \geq 0$ and a fixed cost per claim $c' \geq 0$. So, the expected profit of an insurance company offering contract $z = (\lambda y, y)$ to an agent with ex post risk $1 - p$ is

$$\pi = y - (1 - p)(\lambda y + c') - c$$

**Profit Monotonicity (PM) Assumption:** If two contracts $z_1$ and $z_2$ are chosen in equilibrium and $\lambda_1 y_1 < \lambda_2 y_2$, then $\pi(z_1) \geq \pi(z_2)$.
The (PM) assumption is more general than the standard zero-profit assumption. It holds true even in some competitive models in which the equilibrium involves cross-subsidisation across contracts (Miyazaki, 1977). However, it may be violated in non-competitive models such as Jullien, Salanie and Salanie (2007).

We can now show the following results:

**Lemma 1:** Suppose an agent \( \theta \) chooses the contract \( z_1 = (\lambda_1, y_1, y_1) \) over the contract \( z_2 = (\lambda_2, y_2, y_2) \) that offers higher coverage \( \lambda_2, y_2 > \lambda_1, y_1 \geq 0 \). Then if the agent’s ex post risk under \( z_1 \) is \( 1 - p(z_1) \), it must be true that

\[
1 - p(z_1) < \frac{y_2 - y_1}{\lambda_2 y_2 - \lambda_1 y_1}
\]

**Proof:** See Appendix A.

Intuitively, given risk aversion, if an agent chooses \( z_1 \) (low-coverage) over \( z_2 \) (high-coverage), then it must be true that his accident probability under \( z_1 \) is strictly lower than the per unit premium of the additional coverage offered by \( z_2 \).\(^{27}\) Otherwise, he would be strictly better off taking \( z_2 \) while keeping \( 1 - p(z_1) \). This is a revealed preference argument. Its validity is independent of the market structure, proportional loadings/taxes, or whether some agents go uninsured. For example, it holds in the models of Julien, Salanie and Salanie (2007) and de Meza and Webb (2001) where the positive correlation property breaks. In contrast, in our framework, because some agents (the Os) underestimate their true accident probability, this prediction fails.

**Lemma 2:** Suppose \( z_1 = (\lambda_1, y_1, y_1) = (0,0) \) and \( z_2 = (\lambda_2, y_2, y_2) \) are chosen in equilibrium. If \( c > 0 \) or \( c' > 0 \) or \( c, c' > 0 \), then it may be true that \( 1 - p(C_1) \geq 1 - p(C_2) \). If \( \lambda_2 y_2 > \lambda_1 y_1 > 0 \), then \( 1 - p(C_1) < 1 - p(C_2) \) is always true.

**Proof:** See Appendix A.

\(^{27}\) Equivalently, given risk aversion, if an agent chooses \( z_1 \) (low-coverage) over \( z_2 \) (high-coverage), then it must be true that his expected income under \( z_1 \) is strictly greater than under \( z_2 \).
Intuitively, if all agents purchase some insurance, they all pay the fixed administrative costs through a higher per unit premium. Also, from Lemma 1, if an agent chooses the low-coverage contract, $z_1$, it must be true that the per unit price of the additional coverage offered by $z_2$ exceeds his accident probability under $z_1$. Hence, because insurance companies’ profit on $z_1$ is not less than on $z_2$, the accident probability of an agent choosing $z_1$ must be strictly lower than an agent choosing $z_2$. However, if some agents go uninsured, they do not incur these fixed costs. As a result, although their accident probability is lower than the per unit premium paid by the insured, it is not necessarily lower than the insured’s accident probability because the per unit premium paid by the latter covers both their accident probability and the fixed costs.

Lemma 2 implies that even if we allow for fixed administrative costs standard competitive models of asymmetric information cannot explain the negative correlation between coverage and risk when all agents buy strictly positive coverage. This result allows us to distinguish our model from de Meza and Webb (2001). In Cawley and Philipson (1999) sample all insurees buy some insurance and still there exists negative (zero) correlation between coverage and risk. They also report that the per unit premium falls with coverage even at the largest quantities. More specifically, they find that a fixed administrative cost and a constant marginal cost explain almost all risk-adjusted variation in prices (R-square values close to 1). This suggests the absence of proportional loadings/costs in their sample. Therefore, the de Meza-Webb (2001) model cannot explain the empirical findings of Cawley and Philipson (1999) whereas our model can.

In the monopolistic model of Jullien, Salanie and Salanie (2007), in order to reveal their type and obtain insurance at a lower per unit price, the less risk-averse insurees accept low coverage. Hence, more coverage is associated with a higher per unit price. This prediction is at odds with the Cawley and Philipson (1999) findings. In contrast, our model can explain both the negative correlation between coverage and risk and the fact that insurance premiums display quantity discounts.

If proportional costs/loadings are also involved in the insurance market under study, standard competitive models of asymmetric information can potentially generate negative correlation between coverage and risk even if all agents buy some insurance. In these cases, our model can be distinguished from standard asymmetric information models by using the result of Lemma 1.
**COROLLARY 1**: In the separating equilibria of Propositions 1 and 2 it is respectively true that

\[
(y_R - y_O)/(\lambda_Ry_R - \lambda_Oy_O) < 1 - p(z_O) = 1 - p_o \quad \text{and} \quad \text{(15)}
\]

\[
(y_R - y_O)/(\lambda_Ry_R - \lambda_Oy_O) = 1 - p(z_O) = 1 - p_c \quad \text{(16)}
\]

**Proof**: See Appendix B.

In words, in both equilibria the per unit price of the additional insurance offered by the high-coverage contract is not higher than the Os’ true accident probability under the low-coverage contract. Nevertheless, due to the underestimation of their accident probability, the Os purchase the low-coverage contract although the high-coverage contract is also available.

That is, the violation of the revealed preference condition is consistent with our model but not with standard models. Thus, if the revealed preference condition is rejected by the data, only our model can be consistent with these data and so it can still be empirically distinguished from these models.

However, if the revealed preference condition is not rejected by the data, regardless of the sign of correlation between coverage and risk, our model cannot be distinguished from standard models. Intuitively, proportional loadings increase the marginal price above the actuarially fair price. If these loadings are sufficiently high the marginal price becomes higher than the true accident probability of those buying less coverage (although the actuarially fair marginal price is lower) and so the revealed preference condition is met. Because the prevailing marginal price exceeds the actuarially fair one, some (less risk-averse) agents may choose lower coverage even if their accident probability is higher than that of those buying more coverage (the more risk-averse). As a result, we would observe negative correlation between coverage and risk even if all agents could accurately estimate their accident probabilities. That is, the violation of the revealed preference condition is sufficient but not necessary for the presence of optimism. Because, in general, we cannot determine the effect of proportional costs on the per unit premium of the incremental coverage and the accident probabilities, our test cannot always detect optimism.
Recently, there has been proposed another approach which can be used to empirically distinguish my model from models based on heterogeneous risk preferences (for example, de Meza and Webb (2001)) when the revealed preference test is not decisive. This approach is based on direct measures of risk aversion. Heterogeneity in risk preferences can explain the negative correlation between coverage and risk if the more risk-averse agents: i) purchase more insurance, and ii) take more precautions (lower risks). Fang, Keane and Silverman (2008) study the US Medigap insurance market and report a strong negative correlation between ex post risk and coverage. They also find that those who purchase Medigap tend to be healthier but those who are more risk averse are not particularly healthy. As a result, the authors conclude that heterogeneity in risk preferences cannot explain this negative correlation whereas my approach (overoptimism/overconfidence) can.

In summary, in insurance markets where all agents opt for strictly positive coverage, there are just two events (e.g. life insurance) and the correlation between coverage and risk is negative (or zero) in order to empirically distinguish our approach from standard models, one should take two steps: a) first apply the method of Cawley and Philipson (1999) and try to decompose administrative costs (loadings) into fixed and proportional. If the empirical findings indicate the absence of proportional costs/loadings, then only our approach can be consistent with negative (or zero) correlation between i) coverage and risk and ii) coverage and per unit premium. b) If, however, proportional costs/loadings are detected, then one should apply the revealed preference test. To carry out this test, one needs data on premia, coverage, claims and a number of control variables observed by the insurer (and the econometrician)28. If the revealed preference test is not decisive, and measures of risk aversion are available, then the approach of Fang, Keane and Silverman (2008) can still be used to empirically distinguish my model from models based on heterogeneous risk preferences.

4. Conclusions

Most recent empirical studies on the relationship between coverage and risk find either negative or no correlation. Cawley and Philipson (1999) also report that, in the US life insurance market, insurance premiums exhibit quantity discounts.

28 For more details about this test see Chiappori et al (2006).
Furthermore, Finkelstein and McGarry (2006) analyse the US long-term care insurance market and find positive correlation between perceived risk and coverage and zero correlation between coverage and actual risk.

This paper, by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information, can simultaneously explain all these empirical findings. The more optimistic agents (the Os) underestimate their accident probability both in absolute terms and relative to the less optimistic ones (the Rs) and so purchase less insurance. They also tend to be less willing to take precautions. This gives rise to separating equilibria exhibiting negative or zero correlation both between coverage and risk and between coverage and per unit premiums.

These results have several interesting implications. First, they explain both the negative correlation between coverage and risk and the fact that insurance premiums display quantity discounts reported by Cawley and Philipson (1999). This is a key distinguishing feature of our model. Standard asymmetric information competitive models cannot predict negative or zero correlation in insurance markets where all agents opt for strictly positive coverage, there are just two events (loss/no loss), and no proportional loadings. Thus, the de Meza-Webb (2001) model cannot explain the empirical findings of Cawley and Philipson (1999). Also, in Jullien, Salanie and Salanie (2007) the per unit premium increases with coverage which is at odds with the findings of Cawley and Philipson (1999).

Second, according to Fang, Keane and Silverman (2008), heterogeneity in risk preferences cannot explain the strong negative correlation between coverage and risk in the US Medigap insurance market whereas heterogeneity in risk perceptions (overoptimism/overconfidence) can.

Third, the revealed preference argument of Lemma 1 allows us to empirically distinguish our approach from standard asymmetric information models even in the presence of proportional loadings/taxes. A rejection of this revealed preference argument by the data is consistent with our model but not with standard asymmetric information models. That is, if the revealed preference test fails the data, our model can be empirically distinguished from standard asymmetric information models even if the latter models also predict negative or zero correlation between coverage and risk. However, it should be pointed out that the violation of the revealed preference prediction is a sufficient but not necessary condition for the presence of optimism. In particular, if the degree of optimism is low, this test may fail to detect optimism. If the
revealed preference test is not decisive, and measures of risk aversion are available, then the approach of Fang, Keane and Silverman (2008) can still be used to empirically distinguish our approach from models based on heterogeneous risk preferences.

Fourth, they are consistent with the puzzling empirical findings of Finkelstein and McGarry (2006): the positive correlation between perceived risk and coverage and the simultaneous lack of a positive relation between coverage and actual risk.

Finally, they suggest that a lack of a positive correlation between coverage and risk occurrence can be consistent with the presence of informational barriers to trade in the insurance markets under study.
Appendix A: Proof of Lemma 1

Consider a contract $z' = (\lambda_2 y_2, y')$ with premium: $y' = y_1 + (1 - p(z_i))(\lambda_2 y_2 - \lambda_1 y_1)$. We will show that the agent prefers $z'$ to $z_1$. Notice that if the agent still has ex post risk $1 - p(z_1)$ under $z'$, then he faces the following lottery:

$$L' = (-D_\theta + \lambda_2 y_2 - y', 1 - p; -y', p)$$

(A1)

The expectation of this lottery is:

$$(1 - p)(-D_\theta + \lambda_2 y_2 - y') - py' = (1 - p)(-D_\theta + \lambda_1 y_1 - y_1) - py_1$$

(A2)

Clearly, it is equal to the expectation of the lottery

$$L_1 = (-D_\theta + \lambda_1 y_1 - y_1, 1 - p; -y_1, p)$$

(A3)

which the agent faces under $z_1$. Since $0 \leq \lambda_1 y_1 < \lambda_2 y_2$ and contracts do not overinsure, lottery $L_1$ is a mean-preserving spread of $L'$. Thus, given risk aversion, the agent strictly prefers $L'$ to $L_1$. Furthermore, since under $z'$, if he wishes so, he can choose another $1 - p(z') \neq 1 - p(z_1)$, he strictly prefers $z'$ to $z_1$ and hence to $z_2$ (by assumption, $z_1$ is preferred to $z_2$). However, contracts $z'$ and $z_2$ offer the same coverage. Therefore, since $z'$ is strictly preferred to $z_2$, it must be the case that

$$y_2 > y' = y_1 + (1 - p(z_1))(\lambda_2 y_2 - \lambda_1 y_1) \Rightarrow 1 - p(z_1) < \frac{y_2 - y_1}{\lambda_2 y_2 - \lambda_1 y_1}$$

(A4)

Q.E.D.

Proof of Lemma 2: If $z_1 = (\lambda_1 y_1, y_1) = (0, 0)$, then by Lemma 1 we have:

$$1 - p(z_1) < \frac{y_2}{\lambda_2 y_2} \Rightarrow y_2 - (1 - p(z_1))\lambda_2 y_2 > 0$$

(A5)

In this case, $\pi(z_1)$ is (identically) equal to zero. Therefore,

$$\pi(z_1) = 0 < y_2 - (1 - p(z_1))\lambda_2 y_2$$

(A6)

The expected profit for an insurance company offering contract $z_2$ is

$$\pi(z_2) = y_2 - (1 - p(z_2))(\lambda_2 y_2 + c') - c$$

(A7)
Given (PM), \( \pi(z_1) = 0 \), and the fact that in equilibrium profits cannot be negative, it follows that \( \pi(z_1) = \pi(z_2) = 0 \). Then, using (A6) and (A7) we obtain:

\[
[1 - p(z_2) - (1 - p(z_1))](y_2\lambda_2 + c') > -(1 - p(z_1))c' + c \quad (A8)
\]

Given \( \lambda_2 y_2 > 0 \) and \( c > 0 \) or \( c' > 0 \) or \( c, c' > 0 \), it is clear from (A8) that it may well be true that \( 1 - p(z_1) \geq 1 - p(z_2) \).

If \( \lambda_2 y_2 > \lambda_1 y_1 > 0 \), by Lemma 1 we have:

\[
1 - p(z_1) < \frac{y_2 - y_1}{\lambda_2 y_2 - \lambda_1 y_1} \Rightarrow (y_2 - y_1)(1 - p(z_1))(\lambda_2 y_2 - \lambda_1 y_1) > 0 \quad (A9)
\]

Using the expected profit functions \( \pi(z_i) \), \( i = 1, 2 \), and (5) we obtain:

\[
\pi(z_1) - \pi(z_2) < [(1 - p(z_2)) - (1 - p(z_1))](\lambda_2 y_2 + c') \quad (A10)
\]

Given (PM), (A10) implies \( 1 - p(z_1) < 1 - p(z_2) \). \( \square \)

**Appendix B: Proof of Corollary 1**

Using the zero-profit conditions, we obtain:

\[
\pi_i = p(z_i)y_i - (1 - p(z_i))(\lambda_i - 1)y_i = 0 \quad \Rightarrow \quad \lambda_i = 1/(1 - p(z_i)), \quad i = O, R \quad (B1)
\]

In the separating equilibrium of Proposition 1 we have:

\[
1 - p(z_O) = 1 - p_0 > 1 - p(z_R) = 1 - p_C \quad \Rightarrow \quad \lambda_R > \lambda_O \quad (B2)
\]

Therefore,

\[
\frac{y_R - y_O}{\lambda_R y_R - \lambda_O y_O} = \frac{y_R - y_O}{\lambda_R (y_R - y_O) + (\lambda_R - \lambda_O)y_R} < \frac{1}{\lambda_O} = 1 - p_0 \quad (B3)
\]

In the separating equilibrium of Proposition 2 we have:

\[
1 - p(z_O) = 1 - p(z_R) = 1 - p_C \quad \Rightarrow \quad \lambda_R = \lambda_O \quad (B4)
\]

Therefore,

\[
\frac{y_R - y_O}{\lambda_R y_R - \lambda_O y_O} = \frac{y_R - y_O}{\lambda_R (y_R - y_O) + (\lambda_R - \lambda_O)y_R} = \frac{1}{\lambda_R} = 1 - p_C \quad (B5)
\]

\( \square \)
References


