Stochastic modeling of a two-echelon multiple sourcing supply chain system with genetic algorithm

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Abstract
Purpose – To establish a strategic resource allocation model to capture and encapsulate the complexity of the modern global supply chain management problem.

Design/methodology/approach – A mathematical model was constructed to describe the stochastic multiple-period two-echelon inventory with the many-to-many demand-supplier network problem. Genetic algorithm (GA) was applied to derive optimal solutions through a two-stage optimization process. A practical example and its solution were included to illustrate the GA-based solution procedure.

Findings – The model simultaneously constitutes the inventory control and transportation parameters as well as price uncertainty factors.

Originality/value – The model can be utilized as a collaborative supply chain strategic planning tool to determine efficiently the appropriate inventory allocation and effectively manage the distribution/re-distribution process according to real-time demand.

Keywords Supply chain management, Distribution operations, Inventory control, Sourcing

Paper type Research paper

1. Introduction
Traditional concepts and methods for business management were focused on the optimization of the internal activities in an organization. These methods face a limit on the degree of improving the performance of the entire business system. In the present complex and competitive global marketplaces, effective information sharing and efficient distribution and allocation of inventory are necessary features to be considered and established in order to streamline operations and coordinate activities throughout the supply chain. The necessity to increase allocation efficiency and decrease operational costs is forcing supply chain parties continuously to investigate alternative approaches to improve their entire operations. Optimization models and algorithms, decision support systems, and computerized analysis tools are examples of approaches taken by companies in an attempt to improve their operational performance and remain competitive under growing competition.
Presently, global open market business models support the tendency devoted to the optimization of resource allocation activities with the intense competition among various supply sources. The dependency on one supply source is usually the cause for risk and less efficiency in contract negotiation. In contrast, strategic multiple sourcing generates more competitive environment and thus reduces the likelihood of supply disruption or price escalation. It ensures the steady flow of supplies, and resolves the capacity limitation problem of one-source supply model. Risk factors such as exchange rate volatility, supply disruptions due to labor and political instabilities, and lead-time variability can also be reduced.

The quantitative analysis of the benefit by adopting the multiple sourcing strategies, however, has not received enough attention (Minner, 2003; Steele and Court, 1996). Tullous and Utecht (1992) conducted a research to compare the procurement method of single and multiple sourcing. The interviews from 80 industrial purchasing managers were analyzed to find the decision factors of using single or multiple sourcing. The analysis of variance (ANOVA) method was applied to analyze the uncertainty factors. Their study concluded that more decision support methods needed to be developed in order to enable purchasing managers to make a coherent assessment of whether to use single or multiple sourcing.

Today advanced information technology such as e-business and e-commerce accommodates convenient access to numerous supply sources. To enhance global collaboration and to be able to gain benefits for the entire supply chain system more studies of the multiple sourcing strategy are needed. Our model configures a many-to-many demand-supplier network problem. The model considers inventory control policies and inventory distribution/re-distribution policies of a two-echelon inventory network. Figure 1 depicts the multiple sourcing supply chain system of this study. We present the review of current supply chain management practice and the needs for multiple sourcing in sections 1 and 2. Our mathematical model, which includes various cost elements of a distributed two-echelon supply chain network, is presented in section 3. We address the stochastic nature of the model and the variables and set inventory control and distribution plan over multiple periods. A two-stage solution process is created to first set the control parameters based on long-term statistical data and second to solve day-to-day distribution/re-distribution operations. Section 4 describes the conversion of mathematical expressions into genetic algorithm (GA), which is employed to solve both inventory and distribution planning problems. A numerical example and its solution are illustrated and summarized in section 5. A conclusion is provided in the last section.

2. Strategic supply chain multiple sourcing discussions
There are many aspects to a global supply chain system. Various supply chain management issues have been studied separately and jointly. Some examples of those studies include the facility location and distribution analysis (Daskin, 1995; Bramal and Simchi-Levi, 1997; Current et al., 1997; Klose, 2000; Canel et al., 2001); analysis of inventory control policies (Naddor, 1966); single and multiple echelon inventory systems (Axsater, 2001; Abdul-Jalbar et al., 2003); effective information technology development (Owens and Levary, 2002; Tellefsen, 2002); and many others. Some models have been applied to consider the combined issues, for instance, the single sourcing analysis of integrating inventory and the location of facility (Nozick and Turnquist, 1998, 2001); and the single sourcing with inventory and transportation problem (Das and Tyagi, 1997).
In multiple sourcing models, Kim et al. (2002) considered a supply network consisting of a manufacturer and its suppliers. The problem was presented as a multiple-item single period many-to-one multiple sourcing system. The model gave an optimal solution method to select the supplier. Owing to the nature of the problem described, the inventories at the supply source and the distribution cost were not considered.

Yokoyama (2002) developed a model for a multiple sourcing inventory and distribution system. His model focuses on finding the target inventory and transportation quantity that minimize the total cost of the system by using a random local search method combined with GAs. The model was constructed to provide the advantage in total costs saving to the suppliers. The main consideration of his study is to compare two searching methods to see which one provides better performance in the same searching conditions.

Current supply chain systems promote suppliers and buyers to collaborate their inventory levels and distribution plans in order to reduce the cost of the integrated supply chain system. Our study here extends Kim and Yokoyama’s models. It derives a model and solution to minimize the cost of the entire supply chain.

3. Multiple sourcing inventory-distribution model
In this section we present a mathematical model for a two-echelon multiple sourcing inventory system. The system contains $I$ number of warehouses and serves $J$ number of

![Figure 1. An illustration of a multiple sourcing supply chain network](image-url)
markets. A single item that can be held at any warehouse and be demanded from any market regions is considered. The supply-demand relationship among warehouses and markets are many-to-many. The goal is to determine the target inventories and the allocation quantities from warehouse \( i \) to market \( j \) in order to minimize the expected inventory and distribution costs in a finite planning horizon. Demands at the markets are uncertain while their probability distributions are known. Demand at the warehouse level is propagated from market demand. The problem can be decomposed into three issues:

1. Optimal inventory control policy at the warehouses.
2. Optimal inventory levels at each market.
3. Optimal allocation of warehouse-to-market distribution quantities of each period.

The optimal performance of the global supply chain is directly related to the solutions from those three issues. The mathematical formulation which depicts those objectives and some constraints is illustrated as follows.

The objective function is to minimize the total cost of the supply chain:

\[
\sum_{i \in I} \sum_{j \in J} \sum_{t \in T} \left[ a_{ij}(t) x_{ij}(t) + \frac{x_{ij}(t)}{Q_i} \right] V_{ij}(t)
\]

\[+ \sum_{i \in I} \left[ \sum_{j \in J} w_{ij}(t, S_i) + \sum_{j \in J} m_{ij}(t) \right] \quad (1)\]

s.t.

\[\sum_{j \in J} x_{ij}(t) \leq S_i, \quad \forall i \in I, t \in T \quad (2)\]

\[\sum_{i \in I} x_{ij}(t) \leq M_j, \quad \forall j \in J, t \in T \quad (3)\]

\[x_{ij}(t), S_i \geq 0. \quad (4)\]

The objective function, equation (1), describes the minimization of the total costs of the supply chain, which is composed of the distribution and inventory costs. The first term of the objective function depicts the distribution costs in a finite period \( T \). The number of units distributed from warehouse \( i \) to market \( j \) in period \( t \), \( x_{ij}(t) \), is the decision variable. It is the function of the unit shipping cost from each warehouse to each market, \( a_{ij}(t) \), and the unit price of the item at warehouse \( i \) during period \( t \), \( b_i(t) \). The unit shipping cost and unit price are assumed to be variant among the periods. Due to the instability of other related costs, for instance labor and operation costs, and international business conditions such as exchange rates and tariffs, these incidences cause the unit shipping cost and unit price of the product to be unstable. We may assume that these parameters, \( a_{ij}(t) \) and \( b_i(t) \), are random variables described by some known probability distributions derived from the historical information. For instance, the unit price of the item, \( b_i(t) \), can be assumed to be a random variable with known mean and standard deviation of the normal distribution.
The second term in the first bracket states the total cargo cost for the items distributed from warehouse $i$ to market $j$. Each warehouse specifies its cargo capacity, $Q_i$, which describes the maximum number of products that can be fitted into one cargo. The cargo capacity from each warehouse might be different due to the dissimilarity in product packaging, loading techniques, and the cargo dimensions. With the number of units distributed from warehouse $i$ to market $j$, $x_{ij}(t)$, divided by the cargo capacity, $Q_i$, the calculation with ceiling notation gives the smallest integer greater than or equal to $x_{ij}(t)/Q_i$, which is the value of the total number of cargoes used in distributing the products from warehouse $i$ to market $j$ in that specific period. The fixed cargo distribution cost from warehouse $i$ to market $j$ is the total number of cargoes used multiplied by the cost per cargo from $i$ to $j$, $VC_{ij}(t)$.

Next, the second term in the objective function represents the inventory costs of both warehouses and markets. The first summation stands for all warehouse inventory costs of multiple periods, which is a function of scheduling period $t$ and order levels $S_i$. Here, we use order level $S_i$ as the controllable decision variable of the inventory control policy. We first obtain the optimal scheduling period, $t_o$, for all periods. The best solution of the order level at each warehouse $S_{1o}$ is calculated based on the identical $t_o$. The optimal scheduling period $t_o$ is also used as the baseline for the product allocation periods at the markets and distribution systems. The second summation presents the market inventory costs of multiple periods. Here, the suitable numbers of allocation units, $x_{ij}(t)$, are distributed to the market regions in each period.

Equations (2), (3) and (4) represent constraints. Equation (2) shows that the total number of units distributed from any warehouse to markets in any period will be less than or equal to the level of warehouse inventory, $S_i$. Equation (3) shows that the total numbers of products shipped to any market in any period are less than or equal to the market capacity, $M_j$. Finally, equation (4) is the non-negative condition for all decision variables. Sections 3.1 and 3.2 describe the warehouse and market inventory problems.

### 3.1. Probabilistic warehouse inventory control system

A $(t, S)$ control policy will be used here to illustrate the decision-making process. In a $(t, S)$ inventory control policy, the inventory will be replenished according to scheduling period, $t$, to the inventory ordering level, $S$. The scheduling period is the length of time, measured in time units, between consecutive decisions with respect to replenishments. The objective is to find the optimal scheduling period of the entire system, to, and the order level of each warehouse, $S_{1o}$, which minimize the total inventory costs at the warehouses. The scheduling period is defined as a synchronized system variable. With those conditions, the inventory cost of equation (1) becomes:

$$w \text{INV}_i(t, S_i) = \frac{w}{c_i} \int_{0}^{S_i} (S_i - w d_i) f(w d_i, t) dw d_i + \frac{w}{c_i} \int_{S_i}^{\infty} (w d_i - S_i) f(w d_i, t) dw d_i + w e_{ri}.$$  

(5)

The model assumes an instantaneous supply pattern, in which the warehouse inventory is replenished at the beginning of the period, and then immediately available to the market regions.
Let $f(w_{di}, t)$ be the continuous probability density of the demand, $w_{di}$, for the demand at warehouse $i$ for the scheduling period $t$. Demand in each period could be represented by the probability distribution, such as normal distribution or exponential distribution. From equation (5), the first integration states the carrying cost when the system has inventory left over in the warehouse, which is caused by the difference between the ordered amount and the actual demand. The second term represents the shortage cost when the demand cannot be satisfied from the amount in inventory. The last term indicates the replenishment cost occurring when products are ordered. The model is calculated based on the established variables of the carrying cost per quantity unit per scheduling period, $w_{cci}$; shortage cost per quantity unit per scheduling period, $w_{csi}$; and replenishing cost for each replenishment, $w_{cri}$. We assume that there is no lead-time when the warehouses procure the products from their sources. The solution from a GA process will provide the optimal synchronized scheduling period, $t_o$, to be utilized in all warehouses and their optimal inventory order levels, $S_{1o}$.

3.2 Probabilistic market inventory control system

The market inventory model is a probabilistic system, in which the demand is not known with certainty. Here, we examine the minimum total cost of the overall markets, which are the results from the number of units distributed from warehouse $i$ to market $j$ in each period, $x_{ij}(t)$. Unlike the upper echelon warehouse case, the inventory policy of the lower echelon, the market, is pre-determined by the warehouse level inventory policy. The major consideration at the market level is to determine the optimal numbers of units distributed in each period because of the uncertain demand. The optimal number of units distributed will yield a surplus or shortage at the market place for each period constrained by the objective of minimizing the total supply chain costs. Equation (6) is the total inventory cost of the probabilistic model with the assumption of uniform demand pattern in a period:

$$m_{INV_j}(t) = m_{c_{ij}} \int_0^\infty \left( \sum_{i \in I} x_{ij}(t) - \frac{m_{d_j}}{2} \right) f(m_{d_j}, t) d_{m_{d_j}}$$

$$+ m_{c_{ij}} \int \frac{\left( \sum_{i \in I} x_{ij}(t) \right)^2}{2m_{d_j}} f(m_{d_j}, t) d_{m_{d_j}}$$

$$+ m_{c_{ij}} \int \frac{\left( m_{d_j} - \sum_{i \in I} x_{ij}(t) \right)^2}{2m_{d_j}} f(m_{d_j}, t) d_{m_{d_j}} + m_{c_{ri}}.$$
With the uniform demand pattern, the market inventory is replenished at the beginning of the period and is consumed uniformly during the scheduling period. Since we are using a synchronized scheduling period for all warehouses, all markets receive the products from the warehouses at the same scheduled time. The replenishment from warehouses to market regions is based on market forecasting of the time period \( t \). We assume that all replenishments from various warehouses to a certain market will arrive at the beginning of the scheduling period \( t \). Based on this assumption, the inventory costs, average inventory carrying costs, and shortage costs at the market level are independent of the distribution lead-time. The distribution unit, \( x_{ij} \), is defined based on those assumptions and conditions.

Let \( f(m_{dj}, t) \) be the continuous probability density of the customer demand received at the market \( j \) for the scheduling period \( t \). Equation (6) presents the expected average inventory costs at market \( j \). The first and second terms state the average carrying cost at the market. The third term is the average shortage cost. The last term is the replenishing cost, which occurs in every period when the commodities are ordered. The market unit carrying cost, \( m_{c_{ij}} \), unit shortage cost, \( m_{c_{ij}} \), and replenishing cost, \( m_{r_{ij}} \), are prescribed.

4. Multiple sourcing inventory-distribution system with GA

GA is a stochastic search technique based on the mechanism of natural selection and natural genetics. Beginning with an initial set of random solutions, the GA will converge to the best solution, which represents the optimal or near-optimal solution to the problem (Gen and Cheng, 1997; Michalewicz, 1994). GA has been applied to a number of supply chain management problems as the optimization solution approach in many different configurations (Srinivasan, 2000; Zhou et al., 2002). Conway and Venkataramanan (1994) illustrated the use of GA to solve dynamic facility location problem. Neubauer (1995) dealt with the applicability of GA to production scheduling. Jeong et al. (2002) presented a GA-based computerized system for implementing the forecasting activities required in supply chain management. For our problem, GA is applied to derive optimal solutions through a two-stage optimization process. Figure 2 demonstrates the process framework.

4.1. Stage 1: warehouse inventory control policy process

A warehouse is established primarily for its serving target markets. Therefore, its order level is propagated from the demand patterns of the serving markets. The capacity of a

![Figure 2](Stochastic modeling)
warehouse can be derived from the historical demand data at each warehouse. The objective of the first stage is to set the optimal variables of the synchronized scheduling period for the system and the order level for each warehouse. From the historical demand information that each warehouse collected in the previous $T$ finite periods, we are able to generate the best combination of scheduling period – order level $(t, S)$ inventory control policy using a GA searching engine. Those $(t, S)$ control policy provides the minimum inventory cost for each warehouse.

We present the method of classical GA that uses fixed-length binary strings (as a chromosome) and two operators: binary crossover and binary mutation. The classical GA, which operates on binary strings, requires a modification of an original problem into the appropriate (suitable for GA) form; this includes mapping between potential solutions and binary representation, taking care of decoders or repair algorithms, etc. Therefore, to solve a nontrivial problem using a classical GA, we must transform the problem into a form appropriate for the GA. Figure 3 illustrates a GA approach.

In the GA environment, an optimal solution to the model is called the fitness, which measures the goodness of searching result. In our model, it is the minimum total inventory cost at the warehouse. The decision variable, inventory order level $(S_i)$, is represented in binary strings $v_k$ as a gene (a chromosome). Each chromosome, or a population member, represents a trial solution to the problem. The chromosomes then evolve by crossover and mutation operators, and the optimal solution is reached after several thousand generations. To set up the GA searching process properly, the decision variable, $S_i$, needs to be encoded into a binary string, which in our case the binary representation of a decimal variable of the order level. The range of the variable $S_i$, inventory order level at warehouse $i$, defines the required length of the bit stream $n_k$. That is:

$$n_k \text{ bits}$$

$$(000010100101010 \ldots 01)$$

$v_k$ represents the binary string value in each chromosome $k$, and $k$ is a member of the population. The representation of the length of required bits ($n$) depends on the required precision. For example, the domain of variable $S_i$ is $[l_i, u_i]$ and the required precision is “$\lambda$ decimal point”. The precision requirement implies that the range of domain of each variable should be divided into at least $(u_i - l_i) \times 10^{\lambda}$ size ranges. The required bits (denoted with $n$) for a variable is calculated as follows (Gen and Cheng, 1997):

$$2^{n_i - 1} \leq (u_i - l_i) \times 10^{\lambda} < 2^{n_i} - 1.$$
The mapping from a binary string to a real number for variable $S_i$ is straightforward and computed as follows:

$$S_i = l_i + \text{decimal(substring}_i) \times \frac{u_i - l_i}{2^{n_i} - 1}.$$  

where $\text{decimal(substring}_i)$ represents the decimal value of substring, for decision variable $S_i$. Suppose that the precision is set as no decimal point ($\lambda = 0$). The required bits for variables $S_1$ is calculated as follows:

\[
\begin{align*}
2^{n_i-1} &< (150001 - 0) \times 10^0 \leq 2^{n_i} - 1 \\
2^{17} &< 150001 \leq 2^{18} - 1 \\
\therefore \quad n & = 18
\end{align*}
\]

where $u_1 = 150001$ (warehouse 1 mean demand plus three standard deviations) and $l_1 = 0$. The total length of a chromosome is 18 bits, which can be represented as follows:

$$v_k = \overline{000101000101001110}.$$  

The corresponding value for variable $S_1$ is given in Table I.

This explains the GA representation process. The initial population might be generated randomly with the population size $K$:

$$v_1 = [000010001010011001]$$  
$$v_2 = [001110101010011000]$$  
$$\ldots$$  
$$v_K = [011010101000111001].$$

Each of them can be identified to represent the inventory order level $S_i$:

$$v_1 = [S_i] = \text{[values } S_{i1}\text{]}$$  
$$v_2 = [S_i] = \text{[values } S_{i2}\text{]}$$  
$$\ldots$$  
$$v_K = [S_i] = \text{[values } S_{iK}\text{]}.$$  

<table>
<thead>
<tr>
<th>$v_k$</th>
<th>Binary number</th>
<th>Decimal number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_k$</td>
<td>10001000101110000</td>
<td>140000</td>
</tr>
<tr>
<td>$v_k$</td>
<td>11101001001110110</td>
<td>238838</td>
</tr>
</tbody>
</table>

$S_1 = 0 + 140000 \times \frac{150001 - 0}{2^{18} - 1} = 80110$ units  
$S_1 = 0 + 238838 \times \frac{150001 - 0}{2^{18} - 1} = 136666$ units.

Table I.
Based on the set of members of the solution population, the process of evaluating the fitness of a chromosome is conducted with the following procedures.

**Evaluation procedure:**

1. **Step 1.** Convert the chromosome’s genotype to its phenotype. This means converting binary string into relative real values.
2. **Step 2.** Evaluate the objective function $g(S_{ik})$.
3. **Step 3.** Convert the value of objective function into fitness. $\text{eval}(v_k) = g(S_{ik}), k = 1, 2, \ldots, K$ (population size).

The fitness function values can then be determined as described below:

\[
\begin{align*}
\text{eval}(v_1) &= g(S_{i1}) = w \ \text{INV}_i(t, S_{i1}) \\
\text{eval}(v_2) &= g(S_{i2}) = w \ \text{INV}_i(t, S_{i2}) \\
&\vdots \\
\text{eval}(v_K) &= g(S_{iK}) = w \ \text{INV}_i(t, S_{iK}).
\end{align*}
\]

At this point, the fitness value of each chromosome is revealed and the order of the solutions from the strongest value to weakest value is determined. The next process is selection. The selection procedure is explained by roulette wheel approach, which is constructed as follows.

**Selection procedure:**

1. **Step 1.** Calculate the fitness value $\text{eval}(v_k)$ for each chromosome $v_k$:

\[
\text{eval}(v_k) = g(S_{ik}), \quad k = 1, 2, \ldots, K.
\]

2. **Step 2.** Calculate the total fitness for the population:

\[
F = \sum_{k=1}^{K} \text{eval}(v_k).
\]

3. **Step 3.** Calculate selection probability $p_k$ for each chromosome $v_k$:

\[
p_k = 1 - \frac{\text{eval}(v_k)}{F}, \quad k = 1, 2, \ldots, K.
\]

4. **Step 4.** Calculate cumulative probability $q_k$ for each chromosome $v_k$:

\[
q_k = \sum_{l=1}^{k} p_l, \quad k = 1, 2, \ldots, K.
\]

The selection process begins by generating the random number $z'$ from range $[0,1]$. If $z' \leq q_l$, then select the first chromosome $v_l$. Otherwise, select the $k$th chromosome $v_k(2 \leq k \leq K)$ such that $q_{k-1} < z' < q_k$. The selection process ends in population size ($K$) times, and each time a single chromosome is selected for a new population. Finally, the new population consists of the following chromosomes:
Crossover and mutation are two techniques used in GA search engine. Crossover is based on the cut-point method. For instance, the one-cut-point method randomly selects and exchanges the parts of two parent genes to generate an offspring (for example see Figure 4).

Mutation is used to alter one or more genes. The selected gene will flip whether from 1 to 0 or 0 to 1 (for example see Figure 5).

At this point, the decision variable \([S_i]\) and fitness values \(_{\mu}\text{INV}(t, S_i)\) are evaluated to all \(K\) numbers, and the best individual is determined. This completed one iteration in a GA searching. The procedure will then make the selection and complete more iterations to gain the best fitness solution which will reveal the decision variables, the inventory order level at the warehouses. The GA searching procedure will stop if the solution changes less than 0.01 percent in the last 5,000 generations. At this point, the outcome is supposed to have converged to the optimal solution. The process stated above gives the optimal solution for the inventory order level \((S_i)\) for the specific scheduling period \((t)\). Varying the value of \(t\) and searching for the optimal \(S_i\) will provide us the information to compare the finest combination of \((t, S_i)\). Figure 6 illustrates the flowchart of the first stage, the static inventory setup problem.

4.2. Stage 2: distribution/re-distribution planning process

This stage involves the distribution of the product from source of supply that is the warehouse to the market regions. A market region can get its supply from one or many warehouses. The objective is to find the number of units distributed from each warehouse to market region or regions with the objective of minimizing the total costs to the supply chain. These costs involve the inventory costs at both warehouses and markets over the planning horizon, and the cost of distributing the products.

The units distributed in each route from warehouses to market regions, identified as \(x_{ij}\), are combined to represent a gene string or chromosome. The chromosome values represent an individual member of the population (see Figure 7).

The bound for each gene has to be given based on the constraints. Each gene can have a searching space on its warehouse capacity; for example, \(x_{11}\) may have the unit distribution from 0 to the maximum inventory level of warehouse one. Another
Figure 6.
Flowchart of the first stage GA searching procedure
significant constraint is the total unit distributed from each warehouse, for instance, 
\( x_{11} + x_{12} + \ldots + x_{ij} \), has to be less than or equal to that warehouse’s maximum 
inventory level, which is already specified from stage one. \( x_{ij} \) can be represented and 
encoded to the binary strings as \( x_{ij} \rightarrow 00110010001101000 \). The representation of 
required bits and the mapping from a binary string to a real number for variable \( x_{ij} \) are 
similar to the warehouse inventory stage. All of them can be represented in one 
chromosome as:

\[
X_k = 01011101001100011 \ldots 00110010001101000 \ldots 010100011.
\]

\( X_k \) represents the binary string value in each chromosome \( k \), and \( k \) is the member in the 
population. Then, if the population size is equal to \( K \), the initial population might be 
generated randomly with \( K \) number of chromosomes. Each of them can be identified to 
represent the number of units distribution from warehouse \( i \) to market regions \( j \), \( x_{ij} \):

\[
X_1 = [x_{11}, x_{12}, \ldots, x_{ij}, x_{2j}, \ldots, x_{ij}] = [values \ x_{11}, x_{12}, \ldots, x_{ij}, x_{2j}, \ldots, x_{ij}]_1
\]
\[
X_2 = [x_{11}, x_{12}, \ldots, x_{ij}, x_{2j}, \ldots, x_{ij}] = [values \ x_{11}, x_{12}, \ldots, x_{ij}, x_{2j}, \ldots, x_{ij}]_1
\]
\[
\ldots
\]
\[
X_K = [x_{11}, x_{12}, \ldots, x_{ij}, x_{2j}, \ldots, x_{ij}] = [values \ x_{11}, x_{12}, \ldots, x_{ij}, x_{2j}, \ldots, x_{ij}]_K.
\]

The process of evaluating the fitness of a chromosome, which represents the 
distribution plan, has the following procedure.

**Evaluation procedure:**

1. **Step 1.** Convert the chromosome’s genotype to its phenotype. This means 
   converting binary string into relative real values.
2. **Step 2.** Evaluate the objective function \( h(x_{11}, x_{12}, \ldots, x_{ij}) \), \( k = 1, 2, \ldots, K \) (population size).
3. **Step 3.** Convert the value of objective function into fitness. \( \text{eval}(X_k) = h(x_{11}, x_{12}, \ldots, x_{ij}) \), \( k = 1, 2, \ldots, K \) (population size).

The fitness function values can then be determined as:

\[
\text{eval}(X_1) = h(x_{11}, x_{12}, \ldots, x_{ij}, \ldots, x_{ij})_1 = TC_1
\]
\[
\text{eval}(X_2) = h(x_{11}, x_{12}, \ldots, x_{ij}, \ldots, x_{ij})_2 = TC_2
\]
\[
\ldots
\]
\[
\text{eval}(X_K) = h(x_{11}, x_{12}, \ldots, x_{ij}, \ldots, x_{ij})_K = TC_K.
\]

Then the next process is selection. We use the roulette wheel approach for the selection 
procedure.

![chromosome](Figure 7)
Selection procedure:

(1) **Step 1.** Calculate the fitness value \( \text{eval}(X_k) \) for each chromosome \( X_k \):
\[
\text{eval}(X_k) = h(x_{11}, x_{12}, \ldots, x_{1j}, \ldots, x_{2j}, \ldots, x_{ij}), \quad k = 1, 2, \ldots, K.
\]

(2) **Step 2.** Calculate the total fitness for the population:
\[
F = \sum_{k=1}^{K} \text{eval}(X_k).
\]

(3) **Step 3.** Calculate selection probability \( p_k \) for each chromosome \( X_k \):
\[
p_k = 1 - \frac{\text{eval}(X_k)}{F}, \quad k = 1, 2, \ldots, K.
\]

(4) **Step 4.** Calculate cumulative probability \( q_k \) for each chromosome \( X_k \):
\[
q_k = \sum_{l=1}^{k} p_l, \quad k = 1, 2, \ldots, K.
\]

The selection process, as well as the crossover and mutation processes, is similar to the processes described in section 4.1. The GA searching iterations are terminated when the improvement on the fitness function changes less than 0.01 percent in the last 5,000 generations. The solution stated above gives the optimal or near optimal solution to the total costs of the supply chain. It also reveals the relevant decision variables, the number of products distributed from each supply source to selected market regions. The searching procedure depends strongly on the cost structures and the demand patterns at the market level provided in the model. Moreover, the solution to the total supply chain cost minimization problem can be easily converted to show the service levels or the demand fulfillment level at the market regions. Figure 8 illustrates the flowchart for the distribution planning stage.

This section presented the configuration and conversion of the two-echelon stochastic supply chain management problem to the GA. We will present a numerical example to illustrate this model and explain its solution process in the next section.

### 5. Numerical example and its application

For a common global supply chain network system, market regions are defined all over the world and warehouses are strategically located to stock and supply the demand generated from the markets. The number of warehouses and the market regions are specified for an inventory-distribution problem. In general, the products concerned here are discrete standard products such as home electronics products and standard components or materials, which are easily attained in traditional warehouses. Owing to the simplicity of the products, they are assumed to have the same quality characteristics from each warehouse. With the help of real time information technology, the market consumers are able to examine for the best offering prices. This creates the intense competition among warehouses or suppliers. The competition will force warehouses to offer different prices for different market regions in each period.
Figure 8. Genetic algorithms in distribution stage flowcharts

Stochastic modeling
Since the capacity of each warehouse is relatively stable, the allocation from one source that offers the lowest price to market region might not be feasible. Hence, to benefit the total costs saving in the supply chain, our study will explore and reveal the strategy and the real time adjustment to achieve the minimal cost of the entire supply chain.

### 5.1. Warehouse inventory control analysis

At the inventory stage, we assume that we know the demand history of all warehouses. We use this information combined with some basic forecasting method in order to obtain the initial input for the model. In the following numerical example, the demand at each warehouse is generated randomly following a normal distribution with the mean and standard deviation known. Considering three warehouses, $W_1$, $W_2$, and $W_3$, all offer identical products to the markets. Demand information in 30 consecutive periods ($T$) is generated and calculated by GA to provide the inventory control policies of each warehouse. The input parameters are provided in Table II.

For calculating the optimal solution, the scheduling period ($t$) is specified in discrete time units, $t = 0.5, 1, 2, 3, 4$, which represent weeks, months, or years. The order level, $S_i$, is assumed to be continuous integer values. Five solutions, $(0.5, S_i)$, $(1, S_i)$, $(2, S_i)$, $(3, S_i)$, $(4, S_i)$, from the scheduling periods are compared and the combination, which represents the least inventory costs, will be selected as the inventory control policy of that warehouse. Table III shows the results of 30 periods calculated from GA showing the optimal number of inventory units of warehouse 1. Table IV shows the period-by-period costs and the average total costs of warehouse 1 in 30 periods resulted from GA.

Table V is the summary from calculated inventory policies of all warehouses. The italicized entries indicate the optimal solution for the warehouse inventory control policy.

The optimal solutions from GA clearly result in the synchronized warehouses scheduling period, $t_o = 1$. The optimal order levels for the warehouses are $S_{1o} = 107,384$ units, $S_{2o} = 83,524$ units, $S_{3o} = 134,326$ units.

### 5.2. Distribution planning analysis

Considering four markets, $M_1$, $M_2$, $M_3$, and $M_4$, dispersed around the world. We assume that the demands at the markets are generated randomly regarding to the known normal distribution $(\mu, \sigma)$. The demand of each market is uncorrelated to each other. Unsatisfied demands in each period are assumed to be lost sales. Input parameters are listed in Table VI.

Each warehouse offers a different product price for the market regions mainly because of the difference in warehouse operating and shipping costs. The rates of cargo

<table>
<thead>
<tr>
<th>Inventory</th>
<th>$b_i$ ($)</th>
<th>$w_{cci}$ ($)</th>
<th>$w_{cis}$ ($)</th>
<th>$w_{cri}$ ($)</th>
<th>$\sigma$ (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>50.00</td>
<td>15.00</td>
<td>30.00</td>
<td>1,000.00</td>
<td>100,000</td>
</tr>
<tr>
<td>$W_2$</td>
<td>55.00</td>
<td>16.50</td>
<td>33.00</td>
<td>1,200.00</td>
<td>80,000</td>
</tr>
<tr>
<td>$W_3$</td>
<td>45.00</td>
<td>13.50</td>
<td>27.00</td>
<td>800.00</td>
<td>120,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II. Warehouse inventory input parameters</th>
<th>Product prices ($b_i$) ($)</th>
<th>Unit carrying cost (30 percent) ($w_{cci}$) ($)</th>
<th>Unit shortage cost (60 percent) ($w_{cis}$) ($)</th>
<th>Replenishing cost per period ($w_{cri}$) ($)</th>
<th>Demand (units)</th>
<th>Standard deviation (units)</th>
</tr>
</thead>
</table>
costs, $VC_{ij}$, are derived from actual shipping cost (Maersk Sealand, 2003). Based on the size of the cargo, the size of the product, and the cost per cargo shipped, the unit shipping cost can be computed. Since the rates always fluctuate, the unit shipping cost is varied within a specified range. The prices of the products at the markets are given. The price of the product in each market is different due to the market condition, currency exchange rate, and taxes. Table VII shows the input parameters for the markets.

Using GA to find the optimal distribution/re-distribution planning, the best fitness solutions reveals the minimum total costs, $TC$, to the supply chain and the number of unit distributed, $x_{ij}$. From the calculations, we have $x_{11} = 7,166$, $x_{12} = 13,170$, $x_{13} = 11,015$, $x_{14} = 23,912$, $x_{21} = 69$, $x_{22} = 6$, $x_{23} = 28$, $x_{24} = 27$, $x_{31} = 49,744$, $x_{32} = 25,326$, $x_{33} = 32,370$, and $x_{34} = 26,886$ units. Total inventory cost is $11,204,176.02$. Total distribution cost is $8,859,434.94$ and $TC$ is $20,063,610.96$. The solution was obtained after 532,686 trials. The searching was terminated when the solution has not improved for more than 0.01 percent in the last 5,000 trials. Comparing
to a non-optimal policy of equally divide the demand among all three warehouses, the total supply chain costs are $57,384,862.50. The GA approach calculated the trade-off of all cost parameters and derived the optimal allocation of inventory and distribution plan. For this set of data, the reduction of the total cost is more than 65 percent.
The two-stage approach and the use of GAs to manage the two-echelon multiple sourcing inventory-distribution network system provide a flexible tool to handle market changes and uncertainties. For example, if the shortage costs at markets are considerably high due to opportunity loss, loss of customers, loss of profit, and so on. By modifying the input parameters to our GA model, we will be able to quickly obtain a modified solution to adjust to the market change. Assume that the unit shortage cost at a market is increased to 150 percent of its product price and all other input parameters are stable. Applying the GA search model, from the result we noticed that 

\[
\begin{align*}
\text{mcs}_1 &= $375.00, \\
\text{mcs}_2 &= $405.00, \\
\text{mcs}_3 &= $337.50, \text{ and mcs}_4 &= $382.50; \text{ and the distribution/re-distribution planning is} \\
\end{align*}
\]

\[
\begin{align*}
x_{11} &= 24,420, x_{12} &= 27,818, x_{13} &= 21,372, x_{14} &= 18,339, x_{21} &= 79, x_{22} &= 11, x_{23} &= 73, x_{24} &= 40, x_{31} &= 43,487, x_{32} &= 17,844, x_{33} &= 30,822, \text{ and } x_{34} &= 42,173 \text{ units. Total inventory cost is$11,382,108.27 and TC is $22,085,434.84. Compared to the previous case, the numbers of units distributed are increased because the shortage cost is higher. It is more cost effective to distribute more products to markets comparing to the amount in previous situation. Figure 9 shows the GA trials when converged to the optimal solutions at a unit shortage cost of 150 percent of its product price. The results indicate that most allocations come from \( W_3 \), which its capacity is not enough to cover the suitable units distributed to the market regions. Hence, to obtain the minimum total costs, TC, the products need to be redistributed to the destination markets via \( W_1 \) and \( W_2 \). The calculation from GA provides the optimal number of units shipped
from each warehouse to each market zone. Also, the solution from GA demonstrates the minimum total supply chain costs resulting from inventory and distribution costs.

6. Conclusions
A two-stage solution procedure using GA to the stochastic two-echelon multiple sourcing supply chain system problem was proposed in this study. The mathematical model was developed and the solution procedure was decomposed into a two-stage optimization process. First, the inventory control policies based on static data were set up. Then the distribution/re-distribution plans were derived based on real-time market demand. In the inventory policy setting stage, GA provided the results for minimum inventory cost and inventory control policy at each warehouse. This information was used in the distribution planning stage and the solutions from GA described the distribution/re-distribution pattern with the minimum total supply chain costs. The numerical example illustrated the flexibility to handle many uncertain factors. By adjusting the input parameters and problem description, a practitioner and an advanced analyzer will be able to determine the appropriate inventory allocation and manage the distribution/re-distribution process which will minimize the total supply chain costs under uncertain external conditions.

Further research might focus on using different inventory control policies. The model can be extended to describe more than two-echelon inventory system and to handle multiple items.

References


