A study of wavelet analysis based error compensation for the angular measuring system of high-precision test turntables

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Abstract

An angular measuring system is the most important component of high-precision test turntables; its function and precision determine the turntable’s function and precision. The angular measuring system’s error was considered as a stationary signal in the past. An autocorrelation function and spectrum characteristics of the angular measuring system error are analyzed using the cyclostationary signal theory. The idea that the error in the angular measuring system is nonstationary is first put forward; theory is provided to reconstruct the angular measuring system’s error signal using wavelet analysis. The error signal is reconstructed using one-dimensional Mallat’s algorithm. The standard deviation between the reconstructed and the original signal is much less than the angular measuring system’s accuracy. The reconstruction signal is used to compensate the system error instead of the original error signal; the angular measuring system accuracy is improved. © 2005 ISA—The Instrumentation, Systems, and Automation Society.

Keywords: Wavelet transform; Mallet’s algorithm; Signal reconstruction; Cyclostationary signal

1. Introduction

Cyclostationary signal processing applications have significant advantages over more conventional approaches that treat signals as stationary. Advances in signal processing are achieved by processing signals as cyclostationary. It has been recognized that many random time series encountered in the field of signal processing are more appropriately modeled as cyclostationary, rather than stationary, due to the underlying periodicities in these signals. Cyclostationary signal processing techniques exploit the underlying signal periodicities [1]. It is exhibited as correlations between spectral components shifted by reciprocal of the periods of cyclostationarity (cycle frequencies).

The angular measuring system is the most important component of turntables in inertial test equipment; its function and precision determine the turntable’s function and precision. It attaches importance to research on inertia test equipment. The error in the angular measuring system was considered as a stationary signal in the past, but its statistic characteristic is polyperiodic multiple incommensurate. In fact, it has a cyclostationary characteristic and is a nonstationary signal.

With the development of wavelet transform theory, the wavelet analysis is a powerful signal analysis tool that has been used successfully in many areas for about ten years. The multiresolution analysis is one of the most active branches of the wavelet transform theory, it provides an effective way to analyze a nonstationary signal in modern signal processing [2]. In this paper, an error signal of the angular measuring system is reconstructed using a one-dimensional Mallat’s algorithm. The reconstructed signal is used to compensate for the system error instead of the original.
2. Cyclostationarity and spectral correlation

A random process \( x(t) \) is considered cyclostationary if its autocorrelation function is periodic (one period) or polyperiodic (multiple incommensurate) in time \( t \) for each time shift \( \tau \),

\[
m_x(t) = E\{x(t)\} = m_x(t + k/\alpha),
\]

(1)

\[
R_{xx} = E[x(t + \pi/2)x^*(t - \pi/2)] = R_{xx}(t + k\alpha, \tau),
\]

(2)

where \( \alpha \) is cyclic frequency, \( k \) is an integer, and the "\(*\)" denotes complex conjugation. In general, a signal \( x(t) \) is said to exhibit second-order cyclostationary if there exists a cycle frequency, \( \alpha \neq 0 \), for which the cyclic autocorrelation of \( x(t) \) is defined as the Fourier series of a periodic function [3],

\[
R^a_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t - \tau)e^{-j2\pi \alpha t}dt.
\]

(3)

Based on the cycloergodicity, we can substitute the ensemble average with the temporal average, and another equivalent definition of \( R^a_x(\tau) \) is

\[
R^a_x(\tau) = \langle x(t + \pi/2)x^*(t - \pi/2)e^{-j2\pi \alpha t}\rangle_t,
\]

(4)

where

\[
\langle \cdot \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \cdot dt
\]

denotes time average.

If we remove the conjugation operation "\(*\)" in the above definition, then the above cyclic correlation is called the conjugate cyclic correlation function.

Generally speaking, cyclic frequency \( \alpha \) is usually related to the modulation type, carrier frequency and baud rate of the signal, and different signals have different cyclic frequencies \( \{\alpha\} \). By selecting an appropriate \( \alpha \), we can extract the signal-of-interest from multiple signals and suppress the effects of the other signals, which is often referred to as "signal selectivity" [4]. Because the spectral components spacing at the frequency interval of \( \alpha \) are correlated, the cyclostationary is also referred to as "spectral correlation."

3. Analysis of the angular measuring system error

A rotating transformer and inductosyn in the same axis of the turntable are the angular sensors of the rough and accuracy channels, respectively. The rough channel can test wide scope angles; the accuracy channel can test narrow scope precision angles. The rough and accuracy channels are combined to test the wide scope, high resolution and high precision angles. The system angular resolution is 0.0002° and the accuracy is ±0.0006°. The Inductosyn has 360 pairs of poles and measures 1°. The system error has two parts [5].

3.1. Circuit error of the angular measuring system

In the domain of every 0°–1°, the circuit error is

\[
\Delta \alpha_1 = \frac{1}{2} \Delta \varphi_D - \frac{1}{2} \Delta \varphi_D \cos 2\alpha_D + \frac{1}{2} \Delta \epsilon \sin 2\alpha_D,
\]

(5)

where \( \Delta \epsilon \) is the amplitude error of the inductosyn’s two-phase input voltages; \( \Delta \varphi_D \) is the nonorthogonal error of the inductosyn’s two-phase input voltages; \( \alpha_D \) is the test angle.

3.2. Component error of the angular measuring system

The deviation angle of the angular measuring system display from the theoretical zero point is called the zero-point error. The zero-point error is the key point of inductosyn error. The selected zero point is the beginning point. The zero-point error is represented by accumulation error; it has two parts. The zero-point error produced by the process and the technical deviation of the inductosyn is less than 0.5°,

\[
C = [\sin(16\alpha_D) + 0.5(32\alpha_D)]
+ 0.4\sin(48\alpha_D)
+ 0.2\sin(64\alpha_D)]*0.0001°.
\]

(6)

The other error is caused by installment decen-
where \( K \) is the amplitude caused by decetration and declivity of the rotor and stator; \( m (m=1-720) \) is the number of zero-point errors; \( N (N=720) \) is the number poles that the inductosyn has. So, in the domain of every \( 0^\circ -1^\circ \), the error is

\[
\Delta \alpha_2 = \frac{1}{2} [\Delta Q + \alpha_1(\alpha_D)] + \frac{1}{2} \Delta \beta \sin 2\alpha_D
\]

\[
- \frac{1}{2} \Delta Q \cos 2\alpha_D + \frac{1}{2} \alpha_2(\alpha_D) \cos 2\alpha_D,
\]

(8)

where \( \alpha_1(\alpha_D) = B_i + A_i \); \( \alpha_2(\alpha_D) = B_i - A_i \); \( \Delta Q \) is orthogonal error; \( \Delta \beta \) is amplitude error; \( A_i \) are zero-point errors of phase \( A \), \( A = C_1 + C_2 \); \( B_i \) are zero-point errors of phase \( B \).

The independent variable of the angular measuring system is \( \alpha_D \). \( A_i \) and \( B_i \) are 720 in the domain of \( 0^\circ -360^\circ \), respectively. The error in the angular measuring system is composed of component and circuit error. In the domain of \( 0^\circ -1^\circ \), the error is

\[
x(\alpha_D) = \Delta \alpha = \Delta \alpha_1 + \Delta \alpha_2
\]

\[
= \frac{1}{2} [\Delta \alpha_0 + \alpha_1(\alpha_D)] + \frac{1}{2} \alpha_2(\alpha_D) \cos 2\alpha_D
\]

\[
+ \frac{1}{2} \Delta \gamma \cos (2\alpha_D + \theta)
\]

\[
+ \frac{1}{2} \alpha_2(\alpha_D) \cos 2\alpha_D
\]

(9)

where

\[
\theta = \arctan \left[ -\left( \frac{\Delta \beta + \Delta \epsilon}{\Delta Q} \right) \right]
\]

\[
\Delta \alpha_0 = \Delta \phi_D + \Delta Q; \quad \Delta \gamma = \sqrt{(\Delta \beta + \Delta \epsilon)^2 + \Delta Q^2}.
\]

3.3. Nonstationary analysis of the angular measuring system

An autocorrelation function and a spectrum characteristic of the angular measuring system error are analyzed using the cyclostationary signal theory. The second-order transform of the error signal \( x(\alpha_D) \) is

\[
y_s(\alpha_D) = x(\alpha_D) x(\alpha_D - \tau)
\]

\[
= \left[ \frac{1}{2} [\Delta \alpha_D + \alpha_1(\alpha_D)]
\right.
\]

\[
+ \frac{1}{2} \Delta \gamma \cos (2\pi f_0 \alpha_D + \theta) + \frac{1}{2} \alpha_2(\alpha_D)
\]

\[
\times \cos (2\pi f_0 \alpha_D) \left[ \frac{1}{2} [\Delta \alpha_D + \alpha_1(\alpha_D - \tau)]
\right.
\]

\[
+ \frac{1}{2} \alpha_2(\alpha_D - \tau) \cos 2\pi f_0 (\alpha_D - \tau)
\]

\[
\left. \times \frac{1}{2} \alpha_2(\alpha_D - \tau) \cos 2\pi f_0 (\alpha_D - \tau) \right].
\]

(10)

\( \alpha_1(\alpha_D) \) and \( \alpha_2(\alpha_D) \) are stationary signals, \( \alpha_1(\alpha_D), \alpha_2(\alpha_D), \) and \( \cos(2\pi f_0 \alpha_D) \) are statistically independent signals. So, the correlation function is

\[
R_s(\alpha_D; \tau) = E\{y_s(\alpha_D)\} = \frac{1}{4} [\Delta \alpha_0^2 + R\alpha_1(\tau)] + \frac{\Delta \alpha_0 \Delta \gamma}{4} E\{ \cos (2\pi f_0 (\alpha_D - \tau) + \theta) \}
\]

\[
+ \frac{1}{4} E\{ \alpha_1(\alpha_D) \alpha_2(\alpha_D - \tau) \cos (2\pi f_0 (\alpha_D - \tau)) \}
\]

\[
+ \frac{\Delta \alpha_0 \Delta \gamma}{4} \left[ \cos (2\pi f_0 \alpha_D + \theta) \right]
\]

\[
+ \frac{1}{8} \alpha_2(\tau) \cos (2\pi f_0 (\tau)) + \frac{1}{8} \alpha_2(\tau) \cos \left( 4\pi f_0 \left( \alpha_D - \frac{\tau}{2} \right) \right).
\]

(11)
where \( f_0 = 1/T_0 \) (\( T_0 = 1^\circ \)), \( R_x(\alpha_D; \tau) \) is defined as the Fourier series of periodic function

\[
R_x(\alpha_D; \tau) = \sum_{n} R^\alpha_{\alpha}(\tau) e^{jn\tau}.
\]

The autocorrelation function is

\[
R_x(\alpha_D; \tau) = E[y, \alpha_D] = \sum_{n} R^\alpha_{\alpha}(\tau) e^{jn\tau}.
\]

The idea that the error in the angular measuring system is nonstationary is first put forward. A theory is provided to reconstruct the angular measuring system error using wavelet analysis.

### 4. Mallat algorithm

The multiresolution analysis (MRA) concept was proposed in 1988. Orthogonal wavelet construction and the fast wavelet transform algorithm were given; the algorithm was called the Mallat algorithm [6].

A signal is represented by the sum of its smooth approximation (low pass) and its detailed part (ban pass) in multiresolution analysis theory. The detailed part is still represented by wavelets, whereas the smooth approximation is represented by the dilation and translation of a low-pass scaling function \( \phi(t) \), instead of band-pass functions. They have the following relation:

\[
V_0 \supset V_1 \supset V_2 \supset \cdots \supset V_j,
\]

where \( V_j \) is an approximation space with resolution of \( 2^j \). The detail information lost between \( P_j x \) and \( P_{j-1} x \), denoted by \( D_j x \), can be represented by band-pass wavelets, this can be symbolized as

\[
\ldots V_0 = V_1 \oplus W_1, \ldots, V_j = V_{j+1} \oplus W_{j+1}, \ldots.
\]

The information contained in Eqs. (15) can be shown by the multistage filter banks shown in Fig. 1; the term \( P_j x \) is computed by convolving \( P_{j-1} x \) with decomposition low-pass filter \( h_0(-k) \) and keeping every other sample of the output. Simi-
larly, \( D_1x \) is computed by convolving \( P_{j-1}x \) with decomposition band-pass filter \( h_1(-k) \) and keeping every other sample of the output. Therefore the length of the series in Eq. (16) is the same as that of the original signal [7].

The terms \( g_0(n) \) and \( g_1(n) \) are conjugation transposed matrixes of \( h_0(-k) \) and \( h_1(-k) \), respectively; they are called the reconstruction low-pass and band-pass filter. The decomposition signal is first interpolated. The interpolation decomposition signal is convoluted with the corresponding reconstruction filter. This is Mallat reconstruction algorithm and is shown in Fig. 2.

5. Reconstruction of the angular measuring system using the Mallat algorithm

5.1. Reconstruction of the angular measuring system

The resolution of the angular measuring system is 0.0002°, the error signal of the angular measuring system is reconstructed using a db4 wavelet. The reconstructed signal is used to compensate the system error. The reconstruction steps are

1. The error signal is decomposed using a filter, \( \Delta \alpha \) is filtered using a low-pass filter, and every other sample of the output is kept, so the approximation part of the original signal is obtained. Similarly, \( \Delta \alpha \) is filtered using a band-pass filter and every other sample of the output is kept, so, the detailed part of the original signal is obtained (see Fig. 3),

\[
x_k^{(1)} = \sum_n h_0(n-2k)\Delta \alpha,
\]  

(16)

\[
d_k^{(1)} = \sum_n h_1(n-2k)\Delta \alpha.
\]  

(17)

(2) The two decomposition signals are interpolated. The interpolation results are convoluted using reconstruction low-pass filter \( g_0(n) \) and band-pass filter \( g_1(n) \), respectively. The two reconstructed signals are synthesized to obtain the reconstructed error signal,

\[
y(\alpha_D) = \sum_k g_0(n-2k)x_k^{(1)} + \sum_k g_1(n-2k)d_k^{(1)}.
\]  

(18)

A multiple sampling filter bank is used to reconstruct the error signal. The error signal is decomposed and reconstructed using a one-dimensional Mallat algorithm, this process is a typical two-channel filter bank. The reconstruction mean-square error is

\[
\sigma_e^2 = \frac{1}{N} \sum_{n=1}^N [\Delta \alpha - y(\alpha_D)]^2,
\]  

(19)

where \( \sigma_e^2 \) is mean-square error. The error signal is reconstructed in the domain of 0°–360°. Sixteen

Fig. 2. The reconstruction filter bank of the Mallat algorithm.

Fig. 3. Two channel filter bank.
points are sampled in the domain of 0°–1°, so, \( N \) is equal to 5760 in Eq. (19).

5.2. Simulation

The zero-point error produced by the process and technology deviation of inductosyn is less than 0.5°. It is random error. According to Eq. (6), the simulation error curve is shown in Fig. 4.

The other error is caused by installation decentration and declivity of the inductosyn. According to Eq. (7), the simulation error curve is shown in Fig. 5.

The simulation error signal in the domain of 0°–10° is shown in Fig. 6; the reconstructed and original error signals are shown in Fig. 7. The angular measuring system accuracy is 0.0006°. The standard deviation is 1.3994° \( \times 10^{-16} \), it is less than the angular measuring system accuracy. The reconstructed signal meets the system requirement.

The error signal of the angular measuring system is analyzed using the Fourier transform, it is nonstationary and could not be composed of infinite decomposition harmonics. The signal reconstructed using Fourier’s transform is shown in Fig. 8(b); the difference between the reconstructed and original signal is showed in Fig. 8(c). The resolution of the angular measuring system is 0.0002°, the standard deviation of the two signals is

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**Fig. 4.** Zero-point error caused by processing technique.

**Fig. 5.** Zero-point error caused by installation decentration and declivity.

**Fig. 6.** The error signal of the angular measuring system in the domain of 0°–10°.

**Fig. 7.** The reconstructed error signal using the wavelet.
2.8993° × 10⁻⁴; it is more than the system resolution. However, the standard deviation of the wavelet analysis is much less than that of the Fourier transform, so, wavelet transform is very effective in solving this problem.

6. Conclusion

The error in the angular measuring system error of a pipeline detection robot is analyzed using the cyclostationary signal theory; it is tested as a cyclostationary signal. The idea that the error in the angular measuring system is nonstationary is first put forward. The error signal is reconstructed using a one-dimensional Mallat algorithm. The standard error between the reconstructed and original signal is much less than the angular measuring system’s accuracy. The system resolution is 0.0002°, but the standard deviation between the reconstructed signal using Fourier’s transform and the original signal is more than the system resolution. It is shown that wavelet transform is very effective in reconstructing the error signal. At present, these data could be obtained in the angular measuring system. So, the method could be realized in practice. The reconstructed signal, instead of the original error signal, is used to compensate for the system error; the angular measuring system accuracy is improved.

References