Fatigue crack propagation under non-proportional mixed mode loading

R. Plank\textsuperscript{a,}\textsuperscript{*}, G. Kuhn\textsuperscript{b}

Abstract

Distinguished. An existing precrack will either kink, mode I controlled (tensile mode), or will propagate, coplanarly mode II controlled (shear mode). Shear mode growth will occur if the effective mode II range exceeds the material-specific threshold.

Crack is larger than the under the scanning electron microscope showed that flaws are not the reason for the mode II controlled crack propagation and support the criteria introduced. If the crack opening is large enough, the crack propagation rate is higher for shear-stress controlled crack growth than for normal-stress controlled crack extension, the deviation angle of which is well predictable via the MTS criterion due to Erdogan and Sih [On the crack extension in plates under plane loading and transverse shear. J Basic Engng 1963;85:519–25].

Keywords: mode growth

1. Introduction

Since the first days of experimental fracture mechanics, the majority of investigations have focused on mode I loading where stable crack propagation is coplanar and normal to the
loading direction. In engineering applications, however, components are not subjected to pure mode I loading, but usually to a combination of normal and shear loading (mode II or III), called mixed mode. In principle one has to distinguish between proportional and non-proportional mixed mode loading. The latter appears if the ratio between normal and shear stress changes during the loading process.

The essential results in the field of fatigue crack propagation under cyclic, proportional mixed mode I + II loading are summarized in Bold et al. [1] and Schillig [2,3]. If the crack propagates at a certain deviation angle, it is always mode I controlled and vertical to the maximum normal stress (tensile mode growth). After coplanar crack initiation (shear mode

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$a$</td>
<td>crack length</td>
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<tr>
<td>$A$</td>
<td>elongation</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
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<tr>
<td>$F$</td>
<td>force</td>
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<tr>
<td>$K$</td>
<td>stress intensity factor</td>
</tr>
<tr>
<td>$K_{IC}$</td>
<td>fracture toughness</td>
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<tr>
<td>$\Delta K_{I}^{*}$</td>
<td>cyclic stress intensity range at kinked crack under non-proportional mixed mode ((\Delta \varphi)) loading</td>
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<tr>
<td>$\Delta K_{II}^{\text{th}}$</td>
<td>threshold of mode II controlled fatigue crack growth</td>
</tr>
<tr>
<td>$\Delta K_{\text{th}}$</td>
<td>threshold of fatigue crack growth</td>
</tr>
<tr>
<td>$L_K$</td>
<td>mean grain size</td>
</tr>
<tr>
<td>$N$</td>
<td>number of loading cycles</td>
</tr>
<tr>
<td>$R$</td>
<td>ratio of minimum to maximum cyclic stress</td>
</tr>
<tr>
<td>$R_m$</td>
<td>tensile strength</td>
</tr>
<tr>
<td>$R_{p0.2}$</td>
<td>stress at 0.2% remaining elongation</td>
</tr>
<tr>
<td>$R_Z$</td>
<td>roughness of fracture surface</td>
</tr>
<tr>
<td>$t$</td>
<td>specimen thickness</td>
</tr>
<tr>
<td>$w$</td>
<td>specimen width</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>loading angle</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>crack deviation angle</td>
</tr>
</tbody>
</table>

Only the most important parameters are given in the nomenclature. Later on in this publication, further symbols will be introduced. These symbols may, however, be distinguished from one another clearly owing to the indices and the combination of indices. A superscript asterisk ‘*’ denotes the quantities on a kinked supplementary crack, while subscript roman numerals I and II can be put down to the modes of crack opening. Effective, minimum and maximum quantities of cyclic loading are indicated by ‘eff’, ‘max’ and ‘min’; cyclic loading itself is indicated by ‘cycl’.

Stress intensity factors from static load are marked ‘stat’. Subscript indices ‘c’, ‘th’ and ‘sm’ come from the terms ‘critical’, ‘threshold’ and ‘shear mode’ and describe material-specific quantities. The prefix ‘\(\Delta\)’ stands for a range owing to cyclic loading.
growth) up to 1 mm in length, either crack arrest for small ranges or mode I controlled crack deviation for large amplitudes will be the case.

Even though specimens of very different geometry were used, both shear stress controlled and normal stress controlled crack growth was reported also for purely cyclic mode II loading. Similar to the mixed mode, the mode II controlled growth stops if the ranges are small [4,5], whereas it may stabilize over several millimeters in the case of large amplitudes [6]. Statements vary here, and this leads to the conclusion that there are different reasons, depending on the material, for coplanar growth at high and low $\Delta K_{II}$-values. In his compendium of literature, Liu [7] comes to the conclusion that normal stress controlled crack growth is the rule and is thus the most frequent propagation mode. Shear mode growth develops either if the crack is initiated along a sliding plane of optimum orientation or if the effective shear stress at the crack tip exceeds the cyclic shear yield stress.

In order to rule out friction under alternating mode II loading, Roberts and Kibler [8] superposed a small static mode I load. At relatively small ranges, they first noticed stable, coplanar crack growth up to 50 mm in 1971. Without going into the details of non-proportional superposition of modes, they put this kind of crack propagation down to the alternating load. The crack alternates between two maximum mode I controlled positions and thus runs straight at last. Therefore, the crack should deviate in one direction only under swelling load.

Non-proportional mixed mode was first mentioned by Pook [9] in 1980. Without experimental proof, he predicted that the crack would propagate in a direction in which the mode I range would reach its maximum. Like Roberts and Kibler, Otsuka et al. [10] generated a non-proportional mixed mode I + II loading. For their investigations, they used compact specimens with an internal crack and four-point bending specimens, the precracks of which were opened due to static mode I load and were subject to alternating mode II load. Starting from a certain threshold value of cyclic stress intensity, which strongly depended on the material used, the crack grew in a coplanar manner in the direction of the maximum shear stress. They named this kind of propagation shear mode as well. All in all, they distinguished five possible combinations of crack initiation and stable crack propagation which turn from tensile mode to shear mode with growing mode II stress intensity range. They explained this transition by the different development of the plastic zone at the crack tip when increasing the mode II loading. According to their statements, the static mode I load does not have any influence on crack propagation. It is only necessary to surmount friction on the crack surfaces. They did not make any statements, how the deviation angles in tensile mode were influenced by the static load.

From the investigations made so far one can see that non-proportional superposition of modes clearly affects the propagation behaviour of fatigue cracks. Both the crack growth rate and the direction of propagation are influenced by this. The results obtained, however, vary regarding the way of propagation, deviation angle and growth rates. In the investigation on hand further experiments were done on fatigue crack propagation under non-proportional mixed mode I + II loading. Results show to what extent a static load portion influences the crack deviation angle and the crack growth rate under cyclic loading and which factors are responsible for stable coplanar crack growth.
2. Basic relations

Since under mixed mode loading, kinked crack paths with \( K^*_I \) and \( K^*_II \) at the crack tip may occur (see Fig. 1), criteria must be made available for prediction of the crack propagation direction. One possible criterion goes back to Erdogan and Sih [11] and implies that the crack propagates normal to the maximum tangential stress \( \sigma_{\text{ymax}} \) (MTS criterion). The deviation angle of the supplementary crack can be calculated by

\[
\phi_0 = \arccos \left( \frac{3K^2_{II} + K^2_I + 8K^2_{II}}{K^2_I + 9K^2_{II}} \right)
\]

from the stress intensity factors on the initial crack. After Nuismer [12]

\[
K^*_I = \frac{1}{2} \cos \frac{\phi}{2} [K_I (1 + \cos \phi) - 3K_{II} \sin \phi]
\]

and/or

\[
K^*_II = \frac{1}{2} \cos \frac{\phi}{2} [K_I \sin \phi + 3K_{II}(3 \cos \phi - 1)]
\]

are valid for the stress intensity factors of an infinitesimally short kinked supplementary crack.

In the case of proportional mixed mode loading, the ratio between \( K_I \) and \( K_{II} \) remains constant during one loading cycle, and maximum stress intensity range on the supplementary crack

\[
\Delta K^*_I(\phi_0) = K^*_I_{\text{max}}(F_{\text{max}}) - K^*_I_{\text{max}}(F_{\text{min}})
\]

occurs at an angle \( \phi_0 \) (Fig. 2), for which at the same time the values for \( K^*_II \) disappear. In combination with a statically superposed load, the following kinds of non-proportional mixed

![Fig. 1. K-factors at deviated crack.](image-url)
mode I + II loading can be distinguished from one another:

- cyclic mode II loading is superposed by a static mode I load;
- cyclic mode I loading is combined with a static mode II load;
- cyclic proportional mixed mode loading occurs in combination with a static mode I or mode II load; or
- the cyclic loadings referred to are combined with a static mixed mode load.

The $K^*$-factors on the infinitesimal supplementary crack will then result from the Eqs. (2) and (3) and can be written as

$$K_I^*(\varphi) = \frac{1}{2} \cos \varphi \left( (K_I \text{ cycl} + K_I \text{ stat})(1 + \cos \varphi) - 3(K_{II} \text{ cycl} + K_{II} \text{ stat})\sin \varphi \right)$$  \hspace{1cm} (5)$$

and/or

$$K_{II}^*(\varphi) = \frac{1}{2} \cos \varphi \left( (K_I \text{ cycl} + K_I \text{ stat})\sin \varphi + (K_{II} \text{ cycl} + K_{II} \text{ stat})(3 \cos \varphi - 1) \right),$$  \hspace{1cm} (6)$$

and a principal distribution such as in Fig. 3 will develop.

Fig. 3 shows that, unlike in proportional loading, during one loading cycle different angles occur for which $K_I^*$ becomes maximum while $K_{II}^*$ disappears. They are marked with a '+' sign. In one loading cycle, two extreme angular positions $\varphi_{\text{max}}$ and $\varphi_{\text{min}}$ occur for which the $K^*$ reaches its maximum value under maximum and/or minimum load. This leads to a stress intensity range on the infinitesimally short supplementary crack.
which is generally smaller than the range $\Delta K^*_{I} (\varphi_0)$ of the relating proportional loading.

$$\Delta K^*_{I}(\Delta \varphi) = K^*_{I \max}(\varphi_{\max}) - K^*_{I \max}(\varphi_{\min}),$$

3. Experimental procedure

3.1. Materials

Five different aluminium wrought alloys were used in the trials. They were delivered in 15 mm thick, rolled sheets and did not undergo any further defined heat treatment. Table 1 shows the chemical composition of the alloys used as per the manufacturer specifications.

3.2. Mechanical properties

The material parameters of the different aluminium alloys were determined according to DIN 50125 [14] on flat tension specimens and according to the ASTM standard E647 [15] on compact tension specimens. Owing to the rolling process, the specimen sheet showed
microstructural anisotropy. This was taken into consideration by taking out the specimens in different orientation to the rolling direction, in L- or T- and LT- or TL-orientation in accordance with the material test standard ASTM E399 [16], respectively. For tension tests according to DIN 50145 [17], a 100 kN standard tension testing machine of Carl Schenck AG was used. Trials were done position-controlled on three flat tensile specimen for every direction of removal. The results are summarized in Table 2.

In order to determine the material properties of the Paris relation [18]

\[
\frac{da}{dN} = C(\Delta K)^m,
\]

investigation of crack propagation in mode I on CT specimen was done in accordance with ASTM E647 in air and at room temperature. For calculation of the $K$-factor, the equation given in the ASTM standard

\[
K = \frac{F(2 + \frac{a}{w})}{t^\frac{3}{2}(1 - \frac{a}{w})^\frac{3}{2}} \left[ 0.886 + 4.64 \left( \frac{a}{w} \right) - 13.32 \left( \frac{a}{w} \right)^2 + 14.72 \left( \frac{a}{w} \right)^3 - 5.6 \left( \frac{a}{w} \right)^4 \right]
\]

was used. Both generation of the precrack and investigation of crack propagation in mode I were done on a 50 kN resonance pulsating machine manufactured by Rumul.

During investigation of crack propagation, harmonic loading was applied at constant loading ranges. The possibility of crack closure [19] was to be ruled out by keeping to a stress ratio of $R = 0.6$ for the experiments. At the beginning of all trials, the stress intensity range was set to 4.34 MPa$^\sqrt{m}$. The crack propagation was checked visually and the cumulated number of stress cycles was put down in a protocol each time after the crack had grown 1 mm. The results are summarized in Table 3. The standard values for the fracture toughness $K_{IC}$ were taken from literature.
3.3. Mixed mode I + II loading

In order to investigate mixed mode loading, the compact tension shear specimen (abbreviated to CTS specimen) as recommended in [24] and the relating loading device (see Fig. 4) were used. For variation of the loading angle from 0° to 90° by steps of 15°, the specimen was inserted in the receptacle of the testing machine in different ways. Apart from mixed mode, which occurs at loading angles between 15° and 75°, this provides for generation of pure mode II loading at $a = 90°$ and pure mode I loading at $a = 0°$.

For generation of non-proportional mixed mode loading, an additional apparatus was designed which can be adapted to the mixed mode loading device (see Fig. 5). Depending on

Table 3
Fracture mechanics parameters of aluminium alloys

<table>
<thead>
<tr>
<th>Alloy</th>
<th>$C$ [m/cycle]</th>
<th>$m$</th>
<th>$K_{IC}$ [MPa$\sqrt{m}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA 5083</td>
<td>$5.89 \times 10^{-11}$</td>
<td>3.73</td>
<td>$\approx 41$ [20]</td>
</tr>
<tr>
<td>AA 6010</td>
<td>$8.76 \times 10^{-11}$</td>
<td>3.39</td>
<td>$\approx 35$ [2]</td>
</tr>
<tr>
<td>AA 2017</td>
<td>$3.26 \times 10^{-12}$</td>
<td>4.71</td>
<td>$\approx 48$ [21]</td>
</tr>
<tr>
<td>AA 7075</td>
<td>$1.59 \times 10^{-10}$</td>
<td>3.56</td>
<td>$\approx 26$ [22,23]</td>
</tr>
</tbody>
</table>

Fig. 4. CTS specimen and mixed mode loading device.
the angle at which the apparatus was mounted to the specimen, cyclic loading may be
superposed with a static mode I, mode II or mixed mode load.

For mixed mode and mode II trials, only CTS specimen with the dimension \( w = 100 \) and
thicknesses of 10 and 12 mm were used. The \( K_I \) - and \( K_{II} \)-factors were determined by
application of the approximation formulas

\[
K_I = \frac{F}{wd} \sqrt{\pi a} \cos \frac{\alpha}{2} \left[ \frac{0.26 + 2.85 \frac{a}{w-a}}{1 + 0.55 \frac{a}{w-a} - 0.08 \left( \frac{a}{w-a} \right)^2} \right]^{\frac{1}{2}}
\]

and

\[
K_{II} = \frac{F}{wd} \sqrt{\pi a} \sin \frac{\alpha}{2} \left[ \frac{-0.23 + 1.40 \frac{a}{w-a}}{1 - 0.67 \frac{a}{w-a} + 2.08 \left( \frac{a}{w-a} \right)^2} \right]^{\frac{1}{2}},
\]

which were computed with the help of the finite element method [24]. In the course of the
experimental runs, the range of the stress intensity factor \( \Delta K \), the static stress intensity factor
\( K_{stat} \) and the stress ratio \( R \) were varied. Like the mode I experiments, the trials were done in
air and at room temperature. The cyclic loading \( \Delta F \) and the ratio \( R \) remained constant during
the subsequent crack propagation. In the framework of the study on hand, three specific kinds
of superposition were investigated and evaluated separately from one another (see Fig. 6):

- superposition of cyclic mode II and static mode I (see Fig. 6a);
3.4. Crack length measurement

The length of a crack of coplanar growth under mode I loading can be determined relatively reliably with indirect methods, whereas determination of crack tip coordinates of kinking mode II or mixed mode cracks is rather problematic. The compliance or the electrical potential measuring method are not suitable for determination of the $x$- and $y$-coordinates of the current crack tip, so that information on the deviation angle is lost. Conventionally, measured values for kinked cracks are acquired optically via microscope and a reference grid on the specimen. For determination of the crack tip coordinates, however, this method is very labor-intensive and can be applied only for interrupted test routines. Since no electrical signal is provided in this method, automatic recording of the measured values is not possible. Additional measuring inaccuracies due to different observations led to the development of an automatic optical crack length measuring system based on digital image processing [25]. It localizes the crack tip coordinates of cracks with arbitrary geometry automatically and very exactly and provides for on-line evaluation of the results. Experiments which are to be controlled in dependence on crack length, very often taking several hours, can now be carried out automatically, without interruption and unattended.
3.5. Fractographic and metallurgical analysis

After the crack propagation experiments were completed, the crack surfaces and the crack paths were inspected both visually and with a scanning electron microscope. In order to determine the different structural properties of the alloys used and possible structural changes caused by the loading, polished sections of specimen areas which had not been subjected to loading, as well as sections of the incipient fatigue crack and sections of the supplementary crack formed under non-proportional loading, were inspected under the microscope. The issue of transcrystalline or intercrystalline crack propagation was dealt with by additionally etching the grain surfaces and grain boundaries of the polished sections. With the help of image processing, the grain sizes could be determined by the intercepted-segment method in the direction of rolling ($l$) and transversal to the direction of rolling ($t$) and led to the anisotropy parameter $r_{lt} = l/t$. Together with the grain sizes determined, this value is given in Table 4. Owing to soft annealing, alloy 5083 did not show significant anisotropy, while considerable grain deformation occurred in other alloys.

### Table 4
Grain sizes and anisotropy parameter

<table>
<thead>
<tr>
<th>Alloy</th>
<th>$l$ [μm]</th>
<th>$t$ [μm]</th>
<th>$r_{lt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA 5083</td>
<td>30.9</td>
<td>28.1</td>
<td>1.10</td>
</tr>
<tr>
<td>AA 6082</td>
<td>211.0</td>
<td>141.0</td>
<td>1.30</td>
</tr>
<tr>
<td>AA 6010</td>
<td>426.0</td>
<td>274.0</td>
<td>1.55</td>
</tr>
<tr>
<td>AA 2017</td>
<td>271.0</td>
<td>139.0</td>
<td>1.95</td>
</tr>
<tr>
<td>AA 7075</td>
<td>59.2</td>
<td>28.9</td>
<td>2.05</td>
</tr>
</tbody>
</table>

4. Results and discussion

4.1. Crack propagation under mode I and mode II loading

The results of crack propagation experiments under mode I and mode II loading are shown in Fig. 7. It is obvious that alloy 2017 has the highest resistance to fatigue crack propagation, while the high-strength alloy 7075 is least resistant to fatigue crack propagation. Despite the high $R$ ratio, fatigue crack closure caused by roughness can be noticed in coarse-grained alloys, however it no longer affects crack propagation starting from a range of $\Delta K = 6 \text{ MPa}\sqrt{\text{m}}$. The average deviation angle on a CTS specimen under mode II loading was 70.0°. This corresponds almost exactly to the theoretically expected value of the MTS criterion (Eq. (1)), which predicts 70.5° for pure mode II loading. The kinked fatigue crack propagation under cyclic mode II loading occurs mode I controlled under like proportional mixed mode.
Fig. 7. Crack propagation rates under mode I and mode II loading.

- mode I controlled
- mode II controlled

Fig. 8. Different modes of stable crack growth.
4.2. Crack extension under non-proportional mixed mode I + II loading

4.2.1. Modes of stable crack propagation

Every alloy used showed different, specific behaviour for identical, non-proportional loading configuration. In principle, two modes of propagation may be distinguished (see Fig. 8). The mode I controlled deviation of the crack is called tensile mode, the mode II controlled coplanar crack growth is called shear mode. It can be distinguished from tensile mode not only by the direction of propagation, but also by the strongly zigzag crack path and a higher propagation rate.

The crack surfaces of both propagation modes can be distinguished clearly with the naked eye in their structure. In tensile mode, the crack surface is very fine and shows low roughness, which is similar to that of a precrack initiated under mode I loading. Coplanar crack growth, however, has a rather rough crack surface in all alloys, which also shows a distinct fibrous structure in the direction of crack propagation. Detailed investigation of the crack surfaces with the scanning electron microscope shows that the cause for the different microscopic structure of crack surfaces in tensile mode and shear mode is not the different failure mechanism. Fig. 9 shows the crack surface of a mode I controlled kinked crack after approx. 3 mm growth. Owing to the cyclic as well as the static opening of the kinked crack, crack face interaction was low, and there is no indication of strong deformation on the crack surface. At a higher resolution, striations are clearly visible so that exactly one loading cycle can be allocated to the distance between two striae of the tensile mode crack surface for a certain range of the crack propagation rate. The surface of a mode II controlled crack is levelled out due to permanent friction between the crack surfaces and has a plate-shaped structure. Deformation of the crack surface is so significant (see Fig. 10) that even at 4000-fold magnification, striations are not visible. Voids on the surface may be caused by missing

![SEM micrograph of tensile mode fracture surface.](image)
particles from precipitations which break out of the matrix due to the high shearing forces in mode II (see Fig. 11) and leave small dents on the crack surface.

Figs. 12 and 13 show clearly that the crack course is influenced locally by precipitations, but globally, the crack keeps to its stress-controlled direction. Neither the existing pores nor the line-type structure which originate from rolling can influence the direction of the main crack considerably. At most, the generation of by-cracks as in Fig. 13 is possible by the growing together of voids. Structure etching on polished sections of shear mode and tensile mode cracks prove that crack propagation is transcrystalline in both cases (see Figs. 14 and 15). In
all specimen examined, crack propagation was discovered the course of which was not impaired by grain boundaries.

Stable mode II controlled crack growth can be observed only under the loading combination cyclic mode II/static mode I. Both under cyclic mixed mode I + II loading and superposed static mode I as well as under cyclic mode I loading with superposed static mode II, only tensile mode can be determined as a stable crack propagation mode. Once the crack has kinked, it will not reverse to shear mode any more. Mode I controlled crack deviation can therefore be considered as the stable kind of crack propagation under non-proportional mixed mode loading.
The following interrelationships can be stated between the crack propagation mode observed and the varied parameters in the trials:

- If during one experimental run with a constant ratio of $K_{I_{stat}}/K_{II_{max}}$ the value of cyclic stress intensity at the beginning of the run $\Delta K_{II}(a_0)$ is increased, an abrupt change from tensile mode to shear mode will result starting from a certain mode II range. There is a specific range for every alloy. This threshold value for change from mode I to mode II controlled crack propagation is called $\Delta K_{I_{th_{sm}}}$ in the following.
For $R$-values $\geq 0.6$ coplanar crack propagation is not noticed for all alloys, independently from $\Delta K_{II}(a_0)$ and $F_I_{stat}$, while for lower $R$-ratios both tensile mode and shear mode will occur, depending on the static load portion.

For a stress ratio $R=0.1$, mode II controlled crack growth can be forced for almost all mode II ranges $\Delta K_{II} > \Delta K_{II \text{ th sm}}$ by variation of $F_I_{stat}$. If the cyclic stress intensity at the beginning $\Delta K_{II}(a_0)$ is kept constant at the level of the threshold value $\Delta K_{II \text{ th sm}}$, an increase of $F_I_{stat}$ will lead to a similar behaviour as the increase of $\Delta K_{II}$. From a certain value of $F_I_{stat}$ the mode of the stable crack growth changes from tensile mode to shear mode.

4.2.2. $\Delta K_{II}$-threshold for shear mode growth

Under cyclic mode II loading, the permanent crack surface contact reduces the effective $\Delta K_{II}$-value. If the superposed load is small, the effective cyclic stress intensity will be large enough to make the precrack grow in tensile mode, but it will be too low to force a change in the crack propagation mode. Only if the crack surfaces are separated from one another completely due to static prestress, the effective cyclic stress intensity will suffice to generate shear mode growth, from which the condition

$$\Delta K_{II \text{ eff}} > \Delta K_{II \text{ th sm}}$$

(12)

can be deduced as a compulsory prerequisite for coplanar crack growth. It implies that shear mode can occur only if the effective mode II range $\Delta K_{II \text{ eff}}$ exceeds the material-specific threshold value $\Delta K_{II \text{ th sm}}$. The effective mode II range

$$\Delta K_{II \text{ eff}} = f(\Delta K_{II}, F_I_{stat}, R, a_0, R_Z)$$

(13)

is a function of several parameters. Loading parameters are the nominal mode II range $\Delta K_{II}$, the static load portion $F_I_{stat}$, the $R$-ratio and the precrack length $a_0$. As a material parameter, the precrack roughness $R_Z$ enters the function.

The $\Delta K_{II \text{ th sm}}$-values which resulted from the crack propagation experiments under non-proportional mixed mode loading are much lower for the coarse-grained alloys 2017, 6010 and 6082 than they are for the fine-coarsed alloys 5083 and 7075. For the aluminium alloys used, an interrelation of indirect proportion may be established between the average grain size $L_K = 1/2(l + t)$ and the critical threshold value $\Delta K_{II \text{ th sm}}$ in the form of

$$\Delta K_{II \text{ th sm}} = \frac{c}{\sqrt{L_K}}.$$  

(14)

Behaviour of Eq. (14) corresponds to the Hall–Petch relation [26] and proves that a higher resistance to shear stress-controlled crack growth can be allocated to fine-coarsed alloys. Fig. 16 illustrates this for the materials used. The loading range in which change-over from normal stress-controlled to shear stress-controlled crack growth could be observed is marked with error bargraphs.
4.2.3. $\Delta K^{*}_{I}(\Delta \varphi)$-criterion

Since analyses of crack surfaces and crack paths showed that, generally, cracks propagate stress-controlled for the named mixed mode loadings, the discovered change in crack propagation mode can be described by the two mechanical parameters $\Delta K_{II}$ and $\Delta K^{*}_{I}(\Delta \varphi)$ (see Eq. (7)): under proportional loading, $\Delta K^{*}_{I}$ is always larger than $\Delta K_{II}$, and crack propagation occurs normal-stress controlled at $\varphi_{0}$ in tensile mode. Under non-proportional loading, $\Delta K^{*}_{I}(\Delta \varphi)$ may become smaller than $\Delta K_{II}$, so that fatigue crack propagation takes place either normal-stress controlled or shear-stress controlled.

4.2.3.1. Cyclic mode II and static mode I loading. Experiments showed that coplanar crack propagation is not always the case if the critical threshold value $\Delta K_{II \text{ th sm}}$ is exceeded. Especially if the $R$-ratio is higher or if the static load portion is very low, the tensile mode growth
often occurred so that there must be another criterion in addition to the compulsory prerequisite for shear mode stated in Eq. (12). Fig. 17 shows that the second condition for shear-stress controlled crack growth

$$\Delta K_{II} > \Delta K^{*}_I(\Delta \varphi)$$

(15)

explains the influence of $\Delta K_{II}$, $R$ and $F_{I \text{ stat}}$, which were noticed. If the mode II range on the starter crack $\Delta K_{II}$ exceeds $\Delta K^{*}_I(\Delta \varphi)$, the ratio of $\Delta K^{*}_I(\Delta \varphi)/\Delta K_{II}$ is smaller than 1, and shear-stress controlled growth of the precrack is possible if the condition $\Delta K_{II \text{ eff}} > \Delta K_{II \text{ th sm}}$ is also complied with. Larger mode II ranges require higher static loads to fulfill the condition in Eq. (15). At an $R$-ratio of 0.1, this can still be achieved with the additional device shown in Fig. 5, while for $R=0.6$ starting from a $\Delta K_{II}$-value of $\approx 6$ MPa$\sqrt{m}$, the maximum possible force of 15 kN is no longer sufficient. For smaller mode II ranges, the shear mode criterion can just be achieved, but then the critical threshold value $\Delta K_{II \text{ th sm}}$ is no longer exceeded in most of the aluminium alloys used. This explains why shear mode growth was hardly noticed for high $R$-ratios while for $R=0.1$, the shear mode conditions could be met relatively easily.

4.2.3.2. Cyclic mixed mode $I + II$ and static mode $I$ loading. For superposition of cyclic mixed $I + II$ loading and static mode $I$ load, the criteria of Eqs. (12) and (15) for shear-stress controlled crack propagation can also be applied. The graphs in Fig. 18 show that the maximum static load portion possible is not sufficient to force shear mode even at an $R$-ratio of 0.1. This is the reason why in none of the experiments made, coplanar crack growth up to failure could be observed.

4.2.3.3. Cyclic mode $I$ and static mode $II$ loading. Cyclic mode $I$ loading will not result in a mode II range on the starter crack so that the prerequisites for shear-stress controlled crack propagation cannot be fulfilled (see Fig. 19). Independently from the size of the superimposed static mode $II$ portion, the crack propagates in tensile mode. Static mode $II$ load may influence

![Fig. 18. Dependence of the $\Delta K^{*}_I(\Delta \varphi)/\Delta K_{II}$-relation upon the parameters $\Delta K_{II}$, $R$ and $F_{I \text{ stat}}$ for $\alpha=75^\circ$ ($\Delta K_{II}$ in MPa$\sqrt{m}$).](image)
only the deviation angle and the crack propagation rate, but will not force a change in the crack propagation mode.

4.2.4. Comparison between experiments and $\Delta K$-criteria

Fig. 20 compares the boundary conditions for shear mode and tensile mode determined in experiments and the theoretically derived criteria. The two criteria for mode II controlled crack propagation can be confirmed for all specimen materials. If both conditions were met, only the shear mode could be observed. If only one of the two conditions was not met, the crack propagated in the tensile mode. The diagrams show that there is a specific threshold value $\Delta K_{II_{th_{sm}}}$ for every alloy. As explained earlier, it correlates with the grain size and is the higher the lower the average grain diameter is.

The value range for $\Delta K_{II}$ for which crack propagation is mode II controlled is smaller for fine-grained alloys. In order to meet the requirement $\Delta K_{II} > \Delta K_{I}^*(\Delta \varphi)$, it is not possible on the one hand to reduce the mode II range arbitrarily because below the threshold value $\Delta K_{th}$ the crack will not propagate at all. If, on the other hand, the $K_{II_{max}}$-values exceed a certain value, the crack may propagate in shear mode even though tensile mode would have been expected according to the $\Delta K_{II}$-criteria. For prediction of the crack propagation mode it must therefore be taken into consideration that the $\Delta K_{I}^*(\Delta \varphi)$-criterion of Eq. (15) is valid only in the range between the threshold value $\Delta K_{II_{th_{sm}}}$ specific for the material and the limit of linear-elastic consideration.

Due to the combination of high cyclic loading range and relatively low yield point, visible plastic zones will develop in alloy 5083-W28 in shear mode. Validity of the linear-elastic $K$-concept was, however, verified and confirmed by numerical methods [27]. In the remaining alloys plastification was not observed at the beginning of stable crack propagation so that the conditions of linear-elastic fracture mechanics were fulfilled and non-linear material behaviour are not the cause for coplanar crack growth.

Since in addition to a larger crack opening, higher static load will also lead to higher stress...
4.3. Crack deviation under non-proportional mixed mode I + II loading

In Fig. 21, the values of the registered deviation angles are compared to those expected under consideration of the MTS criterion from Eq. 1 for the different mixed mode ratios at the time of maximum loading \( (K_I/K_{II})_{\text{max}} \).
For mode II loading without superposed static mode I load, the deviation angles measured are slightly smaller than theoretically expected according to Erdogan and Sih. As numerical investigations at the CTS specimen show [29] this is caused by the influence of the starter notch which effects small mode I loading resulting in a cyclic mixed mode instead of pure mode II loading. For small and medium static loads, correspondence with the MTS criterion is very good for all alloys. With growing static load portions, differences become larger because due to plastification, the static mode I portion at the crack tip may become less. This leads to smaller effective mixed mode ratios with larger deviation angles. Influence of the yield stress of the alloys is obvious because the angle difference grows if the yield stress is lower. A good example for this is alloy 5083 which strongly deviates from the theoretical curve for relatively small mixed mode ratios if the specimen is thin. Under cyclic mixed mode I + II loading ($\alpha = 75^\circ$), the measured angles correspond to theory even for higher $\left(K_I/K_{II}\right)_{\text{max}}$ values since for the same mixed mode ratio, lower static load is required than under cyclic mode II loading ($\alpha = 90^\circ$).

When comparing the deviation angles from the experiments of cyclic mode I/static mode II loading with the theoretically expected values after Erdogan and Sih, it turns out that the discrepancies are much larger than for loading types with cyclic mode II portion. In order to
achieve equivalent stress intensity at unchanged specimen geometry and initial crack length, higher forces are necessary under mode II in comparison to mode I. When applying the mixed mode condition, this leads to higher static loads which may cause the alloy to start yielding already at smaller $K_{I\text{max}}/K_{II\text{ stat}}$-ratios. Especially for high-ductile materials, the nominal and the effective mixed mode ratio no longer correspond, and the difference between the theoretical and the actual deviation angle becomes very large. There is, however, a tendency especially for the deviation angles in high-strength specimen materials which corresponds to the theoretical curves.

As long as the conditions of linear-elastic fracture mechanics are obeyed and crack propagation is normal-stress controlled, the MTS criterion applies also to the non-proportional superposition of static mode I and cyclic mode II and/or mixed mode I + II. For non-proportional superposition of cyclic mode I and static mode II, this is only partially valid.

4.4. Crack growth rates under non-proportional mixed mode $I + II$ loading

4.4.1. Cyclic mode $II$ or mixed mode $I + II$ and static mode $I$ loading

Under cyclic mode II loading, an increase of the static mode I portion leads to a significantly higher crack propagation rate at the beginning of crack extension. As a direct consequence, the time of failure shifts towards lower endurance. The correlation between $F_{I\text{ stat}}$ and crack growth shown in Fig. 22 for alloy 7075 can be noticed for all specimen materials in any propagation mode. In case of high static prestress, the number of stress cycles endured until failure were up to 15 times smaller than under pure mode II loading. The reason for the higher crack propagation rates is the separation of crack surfaces by the static mode I load. Friction in the precrack is reduced while the effective cyclic stress intensity at the crack tip increases. With growing static load, the influence becomes ever less because starting from a certain prestress, the precrack is completely open. The $\Delta K^{*}-da/dN$-curve in Fig. 22 shows that the influence is limited to the beginning of stable crack propagation only. At low static prestress, the influence of friction on the growth rates can still be measured clearly. Once the surfaces have worn and the crack has a certain length, the crack propagation rate remains within a narrow range, independent of the size of static load. So, for the velocity of crack

![Fig. 22. Influence of static mode I load on crack growth rates under cyclic mode II and cyclic mixed mode $I + II$ loading.](image)
propagation after the precrack asperity has worn, only cyclic loading is decisive. Influence of the static load can no longer be noticed.

The different crack propagation rates for tensile and shear mode are compared with mode I crack growth in Fig. 23. Under identical test conditions, crack growth starts earlier in shear mode than in tensile mode. When comparing the growth rates of the different propagation modes, it can be stated for every type of alloy that the crack growth rate at identical cyclic stress intensity is higher for shear mode than for tensile mode. Although the straying range of coarse-grained alloys is in general higher than that of fine-grained alloys, the strong variation of the growth rate in shear mode is apparent. During mode II controlled crack growth, the crack surfaces touch each other permanently and lead to discontinuous propagation because newly formed asperities must wear first before the crack front may propagate. Crack propagation in tensile mode, on the other hand, is almost continuous if the crack opening is sufficient. It corresponds to propagation under pure mode I loading so that in the case of crack deviation under non-proportional mixed mode I + II loading, one may assume mode I controlled crack propagation.

Fig. 23. Comparison of the crack growth rates under mode I and in tensile mode, respectively shear mode.

Fig. 24. Influence of static mode II load on crack growth rates under cyclic mode I loading.
4.4.2. Cyclic mode I and static mode II loading

Under cyclic mode I loading, a higher static mode II portion leads to a shift of the growth curves to the right towards higher number of loading cycles (see Fig. 24 on the left). In comparison to smaller mode-II preload an increase in endurance up to a factor of 4 may be achieved, depending on the alloy, if mode II prestress is sufficiently high. With static shear load, crack sliding displacement grows, and the surfaces of the starter crack wedge into one another to a higher extent. This results in higher friction which in turn may delay the start of the fatigue crack growth considerably.

5. Conclusions

Under non-proportional mixed mode I + II loading, two kinds of stable crack propagation may be distinguished, depending on the specimen material and on the type of loading. An existing starter crack will either deviate mode I controlled (tensile mode) or show coplanar, mode II controlled propagation (shear mode). Once the crack has deviated, it can no longer be forced back to shear mode even if static or cyclic loading is increased significantly so that in accordance with literature, normal-stress controlled crack deviation can be considered to be the stable form of crack propagation under mixed mode loading. Crack propagation rates in tensile mode correspond well to those under mode I loading, whereas in shear mode significantly less loading cycles until failure and up to ten times higher growth rates are noticed. Basically, two loading parameters and one material-specific parameter are decisive for the kind of crack propagation. To initiate mode II controlled crack growth:

1. effective mode II range must exceed a material-specific threshold value so that the condition

\[ \Delta K_{II}^{\text{eff}} > \Delta K_{II}^{\text{th sm}} \]

is fulfilled; and additionally

2. the \( \Delta K_{II} \)-value on the starter crack must be higher than the \( \Delta K_{I}^*(\Delta \varphi) \)-range on the infinitesimally small supplementary crack so that as a further requirement,

\[ \Delta K_{II} > \Delta K_{I}^*(\Delta \varphi) \]

is valid.

The model of Roberts and Kibler cannot be complied with because, even under swelling shear stress, crack extension in the direction of the starter crack could be noticed. The theory of threshold value which was first introduced by Otsuka et al., on the other hand, can be confirmed. However, as long as linear-elastic fracture mechanics is valid, their model in which they consider different formation of the plastic zone in front of the crack tip to be responsible for the change in the propagation mode cannot be maintained. Occurrence of normal-stress controlled and shear-stress controlled crack growth can be explained exclusively by linear-elastic parameters.

Any correlation between the threshold \( \Delta K_{II}^{\text{th sm}} \) and the common material parameters such
as the stress at 0.2% remaining elongation, the tensile strength or the mode I fracture toughness cannot be established. However, in accordance with the Hall–Petch relation, the $\Delta K_{th \, sm}$-value behaves indirectly proportional to the average grain size, so that a higher resistance to shear-stress controlled crack growth may be allocated to fine-grained alloys in comparison to coarse-grained alloys.

Analysis of the crack surfaces and crack paths under the scanning electron microscope shows that coplanar mode II crack growth is controlled by the cyclic shear stress. Static normal stresses cannot be the cause locally because pits on the crack surface can be traced back to broken-off precipitations, not to normal-stress controlled void generation. Examination of etched and unetched ground sections of specimen with cracks from shear mode or tensile mode showed that in both cases, crack growth is stress-controlled and transcrystalline. Precipitations and anisotropy have only local and short-term effects on the crack course. Globally, the crack keeps to its stress-controlled direction until failure. For prediction of the deviation angle in tensile mode, the maximum tangential stress criterion (MTS criterion) after Erdogan and Sih can be applied as well for non-proportional superposition of mode I and mode II loading if the validity limits of linear-elastic fracture mechanics are taken into account. It may be assumed that the mixed mode ratio at the time of maximum loading ($K_I/K_{II}$)$_{\text{max}}$ controls the deviation angle.

If crack propagation is mode I controlled, superposition of a cyclic mode II loading with a static mode I portion leads to a smaller deviation angle on one hand and to a higher propagation rate at the beginning of crack growth on the other. If in addition to a cyclic mode I loading, static mode II load is applied, this leads to crack deviation and significant growth delay. The reason for the different growth rates is the reduction or increase of friction by the static load portion.

Regarding failure mechanisms, there are no hypotheses at the moment which explain both the different crack propagation rates under tensile mode and shear mode and the existence of the $\Delta K_{th \, sm}$-value. Intensive material analyses may provide important indications here and improve the present ideas.

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