Reliability Analysis of Power Substation with Common Cause Failure
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Abstract—The power system need to sustain high reliability due to its complexity and security. The reliability prediction method is usually based on independent failure. However, the common cause failure affects many components simultaneously in a system, and it turns out the system collapse seriously in a wide range. Therefore, to improve the reliability of the power system practically, the analysis using the common cause failure is required. This paper describes the common cause failure modeling combined with independent failure. Using the dynamic fault tree, the incorporated independent failure and common cause failure are proposed and analyzed, and it is applied to the power substation in order to examine the method.

I. INTRODUCTION
The power system is required to sustain high reliability due to its complexity and security. The reliability prediction method is usually based on independent failure calculated by FTA (Fault Tree Analysis), RBD (Reliability Block Diagram), Markov chain, FMECA (Failure Mode and Effect and Criticality analysis) etc.[1]. However, in practice, the CCF (Common Cause Failures) occurs to the facilities in power system as well as independent failures in many cases. The common cause failure affects many components simultaneously in a system, and it turns out the system collapse seriously in a wide range. Therefore, to improve the reliability of the power system practically, the analysis using the common cause failure is required.

The CCF is simultaneous failures of two or more components. Conventionally, $\beta$ -factor model and MGL (Multiple Greek Letter) model are used to make the modeling of the CCF [2]. The $\beta$ -factor model is simple method which common cause failure is presented by the ratio of the total failures including dependent as well as independent failures, but does not properly reflect the real system. MGL is an expanded model of $\beta$ -factor which includes multi-factors to analyze the system more accurate, while it is difficult to apply the huge system since the method becomes complex as the number of components increases.

This paper introduces the method to select the fatal CCF out of root causes, which can have the system to fall into failure. Then the selected fatal CCF is accurately computed by DFT (Dynamic Fault Tree), which has the additional virtual state to present the CCF so that make the calculation of the CCF simple. The system is modeled by MCS (minimal cut sets), and in this paper, it is proposed that MCS and root causes are presented by matrices and used to know what root cause affects the MCS. The matrix of the critical root cause is thereafter produced to find hazard root causes. The simple example is described to explain the availability of the proposed method. Also, DFT and the matrix of the critical root cause are used to analyze the power substation system.

II. SELECTION OF HAZARD ROOT CAUSES
Root Cause is defined by the source of CCF, such as lightning, weather condition, human error, etc. The power system is apt to be exposed in many root causes, but it is impossible to consider all of the root causes. The critical root causes are the causes which bring the system failure, and should be selected for protecting system. For evaluating the critical root causes in the power system, the system and root causes are properly defined preferentially. These definitions are used to calculate CCF.

A. Matrix of minimal cut set
The power system consists of many components, which can be represented by MCS (Minimal Cut Sets). The MCS is a useful method to evaluate reliability of the systems. It can easily show how system falls into failure, and also CCF can be analyzed easily using the merit of MCS. MCS can be presented by a matrix and defined as

$$M = [m_{ij}]$$

$$m_{ij} = \begin{cases} 1 & \text{iset includes} \ j \ \text{components} \\ 0 & \text{iset does not include} \ j \ \text{components} \end{cases}$$

when the system has $j$ components and is divided to minimal cut sets with $i$ sets [4][6]. In the MCS matrix $M$, row means minimal cut sets in the system and columns means components in the corresponding minimal cut set. For example, if the $m_{i1}$ has the value of 1, the first minimal cut set includes the first components. The independent failure of the component is also defined by the column vector of $F$

$$F = [f^N]$$

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\[ f_j^{\text{IN}} \equiv \text{independent failure of } j \text{ component} \quad (4) \]

It assumes that independent failure rate is constant.

**B. Root Causes Matrix**

The root cause is the source of the common cause failure. For example, the nature disaster or improper operation can induce the failure of components simultaneously. These are called root causes. In the same manner as construction of MCS matrix \( \mathbf{M} \), the root cause matrix \( \mathbf{R} \) is defined by

\[
\mathbf{R} = [r_{jn}] \quad (5)
\]

\[
r_{jn} = \begin{cases} 
1 & j \text{ component influences } n \text{ root cause} \\
0 & j \text{ component does not influence } n \text{ root cause} 
\end{cases} \quad (6)
\]

In the root cause matrix \( \mathbf{R} \), row means root causes affecting the system and column means components in the system. Similarly in the case of MCS matrix, if the \( r_{11} \) has a value of 1, the first roots cause influences the first component. The matrix \( \mathbf{R}^{\text{CC}} \) of the common cause failures which result from root cause is defined similarly as the independent failure vector by

\[
\mathbf{R}^{\text{CC}} = [r_{jn}^{\text{CC}}] \quad (7)
\]

\[
r_{jn}^{\text{CC}} \equiv \text{the failure rate of } j \text{ component by } n \text{ root cause} \quad (8)
\]

It assumes also that common cause failure rate is constant same as the independent failure.

**C. Selection of the hazard root causes**

The MCS matrix \( \mathbf{M} \) and root cause matrix \( \mathbf{R} \) are incorporated to produce the new matrix that explains the common cause failure.

\[
\mathbf{MR} = \mathbf{M} \cdot \mathbf{R} = [mr_{jn}] \quad (9)
\]

This matrix \( \mathbf{MR} \) is called the fatal root cause to MCS. The matrix \( \mathbf{M} \) shows the relationship between minimal cut set and components, and the root cause matrix \( \mathbf{R} \) the relationship between components and root causes, and therefore, as a result of the Eq. (9), the matrix \( \mathbf{MR} \) (Fatal root cause to MCS) explains the relationship between the minimal cut sets and root causes. The \( \mathbf{MR} \) means how many components in a minimal cut set are influenced by a root cause. For example again, if the \( mr_{1j} \) in \( \mathbf{MR} \) has the value of 1, the first component in the first minimal cut set is affected by the first root cause.

The summation of the value of the row in the matrix \( \mathbf{M} \), which is defined previously, has the information about how many components are in a minimal cut set. Since the MCS matrix \( \mathbf{M} \) has only 0 or 1, the sum of elements in each row in the matrix is same to the number of component in a MCS, and it can be represented by the \( \mathbf{S} \) matrix as follows,

\[
\mathbf{S} = [s_j] \quad (10)
\]

\[
s_j = \sum_{k=1}^{n} m_{jk} \quad (11)
\]

Now, the matrix of critical root cause, \( \mathbf{R}^{\text{CR}} \), is defined with using the predefined \( \mathbf{MR} \).

\[
\mathbf{R}^{\text{CR}} = [r_{jn}^{\text{CR}}] \quad (12)
\]

\[
r_{jn}^{\text{CR}} = \begin{cases} 
1 & s_j - mr_{jn} = 0 \\
0 & s_j - mr_{jn} \neq 0
\end{cases} \quad (13)
\]

The \( \mathbf{R}^{\text{CR}} \) represents that which root cause affects each of the MCS. If an \( r_{jj}^{\text{CR}} \) element of matrix \( \mathbf{R}^{\text{CR}} \) is 1, the third MCS is affected by the second root cause.

**III. EXAMPLE OF THE CCF MODEL**

To illustrate the procedure of the previous section, the simple example of the bridge structure is adopted as Fig. 1, and the MCS of this structure can be depicted by Fig. 2. Using the Eq. (1), (2), the MCS matrix \( \mathbf{M} \) can be constructed as Eq. (14).

\[
\mathbf{M} = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 1
\end{pmatrix} \quad (14)
\]

The first row in Eq. (14) means the first minimal cut set in Fig. 2 includes component 1, 2. The matrix \( \mathbf{S} \) also has the values as...
It assumes that there is two root causes. First root cause affects to component 2, 4 and second root cause affects component 1, 4, 5. Then the root cause matrix \( R \) can be expressed

\[
R = \begin{pmatrix}
0 & 1 \\
1 & 0 \\
0 & 0 \\
1 & 1 \\
0 & 1
\end{pmatrix}
\]

(16)

The matrix \( MR \) is the production of the matrices \( M \) and \( R \).

\[
MR = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 & 1 \\
1 & 0 \\
0 & 0 \\
1 & 1 \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
1 & 1 \\
1 & 1 \\
1 & 3 \\
1 & 1
\end{pmatrix}
\]

(17)

Then the \( R^{CR} \) is represented by matrices \( MR \) and \( S \).

\[
R^{CR} = \begin{pmatrix}
0 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0
\end{pmatrix}
\]

(18)

The \( r_{32}^{CR} \) in matrix \( R^{CR} \) has only value 1. That means all the third minimal cut set components, 1, 4, and 5, are affected by second root cause. To increase the reliability, the second root cause should be excluded from the system.

IV. DYNAMIC FAULT TREE ANALYSIS WITH CCF

If three components A, B, C are made up of a system with parallel connection as shown Fig. 3[5], then the total failure of component A can be generally expressed by

\[
A_T = A_I + C_{AB} + C_{AC} + C_{ABC}
\]

(19)

where,

- \( A_T \): Total failure of component A
- \( A_I \): Independent failure of component A
- \( C_{AB} \): Common cause failure of component A & B
- \( C_{AC} \): Common cause failure of component A & C
- \( C_{ABC} \): Common cause failure of component A, B & C

If the components B and C follow the same manner of component A, the fault tree is expressed by Fig. 4. If the relationship of components is aptly defined and the number of CCF is limited to relatively be small, this method is simple to express the CCF failure. However, the number of component and CCF increases, the number of CCF elements will also grow exponentially and it is not possible to manipulate in this manner.

**Dynamic Fault Tree with CCF**

Fault trees analysis has been widely used for reliability analysis. A weak point of traditional fault tree analysis is that if the number of components and CCF is increased, there is no way to analyze it. This weak point can be improved by Dynamic fault tree such as SEQ (Sequence Enforcing Gate), FDEP (Functional Dependency Gate), PAND (Priority And Gate), CSP (Cold Spare Gate) [7].

![Fig. 3. Traditional fault tree](image)

![Fig. 4. Traditional fault tree with CCF](image)
This paper uses the FDEP which has single trigger input and one or more dependent basic events [8]. The characteristic of FDEP is described in Table I.

<table>
<thead>
<tr>
<th>Trigger</th>
<th>Output</th>
<th>Dependent component</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>False</td>
<td>False</td>
<td>True or False</td>
</tr>
</tbody>
</table>

The example of the FDEP method is illustrated by Fig. 5. The Fig. 5(a) describes the example system of section III based on independent failure. On the other hand, the Fig. 5(b) describes the result of Eq. (18), which is considering the root cause and the final critical root cause as occurred in the third minimal cutest.

Generally, the reliability of parallel structure of order \( n \) is calculated as

\[
G_R = 1 - \prod_{i=1}^{n} (1 - R_i) = \prod_{i=1}^{n} R_i
\]  

(20)

In Fig. 5(a), the reliability of this parallel system for \( i \)-th MCS is calculated by

\[
G_i = \prod_{k=1}^{j} \exp[-f_k^{IN} \cdot m_k \cdot t] \quad \text{if } r_{ik}^{RC} = 0
\]  

(20)

where, \( t \) is operating times.

The reliability including FDEP for \( i \)-th MCS which is exposed to a \( n \)-th root cause, as depicted in Fig. 5(b), is estimated by

\[
G_{in} = \prod_{k=1}^{j} \left\{1 - \exp[-f_k^{IN} \cdot m_k \cdot t] \left(2 - \exp[-r_{ik}^{CC} \cdot m_{ik}^{CC} \cdot t]ight)\right\}
\]  

(21)

if \( r_{in}^{CR} = 1
\]

According to Fig. 5(b), the CCF plays a role of trigger in FDEP and the minimal cut set 3 is dependent basic event.

V. CASE STUDY

The methods of \( R^{CR} \) and MR are used to set up the root causes modeling. These methods applied to the power substation to analyze the common cause failure. The power substation which is located at Gunbuk 154kV substation in South Korea is summarized by Fig. 6. The name and failure rate of components are described in the Table II. In Table II, the reliability is based on 10,000 operating time. In Table III, root causes and components influenced from them are assumed. It also assumes that the components affected by a root cause have same failure rates. The MCS of the power substation is constructed in Fig. 7. It assumes that the series connection of components A6, A7, and A7 is combined into A678 for simple calculation. In the same way, components B678, D123, E123, F123, and G123 are also defined. Using Fig. 5, the MCS matrix \( M \), the root cause matrix \( R \) and the critical root cause matrix \( R^{CR} \) are expressed as Eq. (22), (23) and (24), respectively. From Eq. (24), it can be seen that the first and the third root causes influence the system seriously.

As the traditional method, the independent failure rate is used to calculate the system reliability. The reliability for each minimal cutest is estimated by Eq. (20), only when the independent failure rate is used,

\[
\left\{\begin{array}{l}
G_1 = 1 - (1 - \exp[-1.2715 \times 10^{-5} \times 1 \times 100000])^5 \\
G_2 = 1 - (1 - \exp[-2.2896 \times 10^{-6} \times 1 \times 100000])^5 \\
G_{12} = 1 - (1 - \exp[-3.999 \times 10^{-6} \times 1 \times 100000])^5
\end{array}\right.
\]  

(25)

The reliability without CCF is
Reliability = $\prod_{k=1}^{12} G_k = 0.8011$  \hspace{1cm} (26)

Table II

<table>
<thead>
<tr>
<th>Code</th>
<th>Component failure</th>
<th>Failure rate $10^{-6}$</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1, B1</td>
<td>GCB</td>
<td>1.2715</td>
<td>0.8807</td>
</tr>
<tr>
<td>A2, B2</td>
<td>DS</td>
<td>2.2896</td>
<td>0.9774</td>
</tr>
<tr>
<td>A3, A6</td>
<td>GCB</td>
<td>1.2715</td>
<td>0.8807</td>
</tr>
<tr>
<td>A5, B5</td>
<td>GCB</td>
<td>1.2715</td>
<td>0.8807</td>
</tr>
<tr>
<td>A7, B7</td>
<td>Scott Tr.</td>
<td>8.3405</td>
<td>0.9199</td>
</tr>
<tr>
<td>D1, D3</td>
<td>DS</td>
<td>2.2896</td>
<td>0.9774</td>
</tr>
<tr>
<td>E1, E3, F1, F3</td>
<td>GCB</td>
<td>1.2715</td>
<td>0.8807</td>
</tr>
<tr>
<td>G1, G3</td>
<td>GCB</td>
<td>1.2715</td>
<td>0.8807</td>
</tr>
<tr>
<td>C1, C2, C3</td>
<td>BUS</td>
<td>3.999</td>
<td>0.9671</td>
</tr>
</tbody>
</table>

Table III

<table>
<thead>
<tr>
<th>$r_{C_i}$</th>
<th>The influenced components</th>
<th>Failure rate $10^{-6}$</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{C_{i1}}$</td>
<td>A4, A5, B4, B5</td>
<td>2.2896</td>
<td>0.9977</td>
</tr>
<tr>
<td>$r_{C_{i2}}$</td>
<td>A1, A3, A5</td>
<td>1.2715</td>
<td>0.9873</td>
</tr>
<tr>
<td>$r_{C_{i3}}$</td>
<td>C2, B678</td>
<td>3.9991</td>
<td>0.9960</td>
</tr>
<tr>
<td>$r_{C_{i4}}$</td>
<td>B5, G123</td>
<td>7.2749</td>
<td>0.9298</td>
</tr>
</tbody>
</table>

As above results, the sever influence factors to the system are the first and the third root causes. If these root causes occur, the sixth and tenth minimal cutest are influenced and system falls in failure. Using Eq. (21), FDEP can be calculated by,

$$G_{k} = \prod_{k=1}^{19} \left[1 - \left( 1 - \exp[-f_k^{D} \cdot m_k \cdot f] \right) \left( 2 - \exp[-f_k^{C} \cdot m_k \cdot f] \right) \right]$$ \hspace{1cm} (27)

$$= 0.9124$$

$$G_{k} = \prod_{k=1}^{19} \left[1 - \left( 1 - \exp[-f_k^{D} \cdot m_k \cdot f] \right) \left( 2 - \exp[-f_k^{C} \cdot m_k \cdot f] \right) \right]$$ \hspace{1cm} (28)

$$= 0.8802$$

The final reliability considering CCF is obtained as

$$\text{Reliability}_{CCF} = \prod_{k=1}^{10} G_k \cdot G_{6,1} \cdot G_{10,3} = 0.6697$$ \hspace{1cm} (29)

The results of the reliability with CCF is low than that of the reliability without CCF. This means that if only the independent failure rate is used for calculating the reliability, the result tends to overestimate the system reliability.

VI. CONCLUSION

This paper analyzes the power system reliability with common cause failure. To consider the common cause failure in system, the matrices are defined to express the relationship among minimal cutest, components, root causes and critical root causes each other. The DFT is adopted for a method to calculate the common cause failure. Finally, the reliability with CCF is compared to reliability without CCF, and the result shows that the reliability concerning the independent failure only is apt to conduct overestimation of the system reliability comparing the case of CCF considered.

REFERENCES


\[
\begin{align*}
M &= \begin{pmatrix} 
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{pmatrix} \\
(22)
\end{align*}
\]

\[
R = \begin{pmatrix} 
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \\
(23)
\]

\[
M^* = \begin{pmatrix} 
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \\
(24)
\]

\[
R^{x*} = \begin{pmatrix} 
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix} \\
(25)
\]

Fig. 7. MCS of power substation