Reliability evaluation and enhancement of distribution systems in the presence of distributed generation based on standby mode

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Abstract

This paper describes an analytical methodology for reliability evaluation and enhancement of distribution system having distributed generation (DG). Standby mode of operation of DG has been considered for this purpose. Reliability of the composite distribution system has been optimized with respect to failure rate and repair time of each section of the distribution system subject to constraints on (i) failure rates/repair times and (ii) customer and energy based reliability indices i.e. SAFI, SAIDI, CAIDI and AFNS. The objective function includes (i) cost of modification for failure rates/repair times (ii) additional cost of expected energy supplied by DG. Differential evolution (DE), Particle swarm optimization (PSO) and coordinated aggregation based PSO (CAPSO) have been used to develop computational algorithms. Developed algorithms have been implemented on a sample distribution system.

Keywords: Distribution system, Reliability indices, Failure rate, Repair time, Differential evolution, PSO

1. Introduction

Distribution systems were not originally designed to accommodate generation in itself. Hence increasing penetration level of DG is causing changes in the planning, operation and maintenance of distribution systems. Presence of DG in a distribution network affects network planning, operation and maintenance, auxiliary services, quality of service and regulatory aspects [1–4]. Ackerman et al. [5] have analyzed the term DG in detail. Usually in reference to this present paper, DG will be considered as electricity generation systems connected to distribution networks, characterized by their small rating and located near connection points. Distribution system operators (DSOs) may have their own DG or may encourage large consumer to install DG which is owned and controlled by these customers. Availability of such customer owned and controlled DG may not be high, but may be of great help in improving the reliability indices of the distribution network.

The reliability of a distribution system may be increased by modifying failure rate and repair time of each section of the network. Such modifications may require additional investments which in the presence of DG may be mitigated. This will result in annual savings. On the other hand cost per unit energy obtained form DG may be high. Further the traditional reliability indices covered sustained interruption durations. The time necessary to start up the DG should be taken into account for the reliability evaluation of distribution system. If this time is sufficiently short the customers suffer a momentary interruption, while, if not, they suffer a sustained interruption.

Various reliability optimization algorithms have been developed by reducing failure rate and repair time of each section of the distribution systems. Sallam et al. [6] presented a methodology for determination of optimal reliability indices for distribution system using gradient projection method. Chang and Wu [7] developed a methodology for obtaining optimal reliability design of electrical distribution system using a primal dual interior point algorithm. Chowdhury and Custer [8] developed a value based approach for designing urban distribution system. Bhowmik et al. [9] described a distribution network planning strategy by considering a nonlinear objective function with linear and non linear constraints for radial distribution system. Su and Lii [10] obtained optimum failure rates and repair times for each section of distribution system using modified genetic algorithms. Particle swarm optimization along with decomposition has been used for reliability optimization of radial distribution systems by Arya et al. [11]. Bae and Kim [12] developed methodology for evaluating reliability of distribution system having DG in operation mode. Bae et al. [13] developed an optimal operating strategy for distributed generation considering hourly reliability worth. Louit et al. [14] presented a
simple technique for obtaining optimal interval for major preventive maintenance for distribution system without DG. Costa and Matos [15] described various modeling aspects for assessing the effect of microgrids to the reliability of distribution network. Trebolle et al. [1] proposed a market mechanism, referred to as reliability options for distributed generation (RODG), which provides distribution system operators with an alternative to the investment in new distribution facilities.

In view of the above discussion this paper presents a new methodology for obtaining optimal failure rate and repair time of each section of a distribution system in the presence of distributed generation which is specifically customer owned and controlled. It is assumed that utility has to pay additional price for each unit charged. An objective function which reflects cost of modification of failure rates, and repair rate and cost of energy supplied from DG has been constructed and minimized subject to reliability constraints.

2. Reliability modeling aspects with distributed generation

Distributed generation which is owned and controlled by customer or any other agency may be used as standby units for reliability enhancement at specific load points. This application is similar to one used in transferring load from one substation to another by disconnecting feeders at their normal source (in the event of failure of a section) and closing the open point. Usually this is done in the event of a failure causing loss of supply from the source substation and it is not normal practice to parallel sources. Now in the changed scenario of deregulated environment, the transfer of load at a load point may be partial or total according to the capacity available of DG. If DG capacity is sufficient to meet the load at a load point in the event of failure of supply from substation then the situation at load point is represent as shown in Fig. 1a.

Further it is to be understood that load is usually connected from source via various distributor segments. By manual switching the load may be shifted to DG in the event of trouble from source side. The average time to carry out the switching operation is assessed as ‘s’ hours. The equivalent reliability diagram for the system is shown in Fig. 1b in which source side, and DG are directly connected in parallel but placed in series with important switching element with failure frequency \( \lambda_{sw} \) and restoration time ‘s’. Reliability network of Fig. 1b may reduced by the application of series–parallel approximate formulas [16]. The system of Fig. 1b is reduced to an equivalent network \( (\lambda_{req}, r_{eq}) \) shown in Fig. 1c using following relations

\[
\lambda_{eq} = \lambda_s \cdot \lambda_{dg} \cdot (r_s + r_{dg}) + \lambda_{sw}
\]

\[
r_{eq} = \frac{\lambda_s \cdot \lambda_{dg} \cdot r_s \cdot r_{dg} + \lambda_{sw} \cdot s}{\lambda_s \cdot \lambda_{dg} \cdot (r_s + r_{dg}) + \lambda_{sw}}
\]

where \( \lambda_{eq} \) is the equivalent failure rate; \( r_{eq} \) the equivalent interruption duration; \( \lambda_s \) the represents total failure rate up to load point from substation supply; \( r_s \) the represents average interruption duration from substation supply; \( \lambda_{dg} \) the failure rate of DG; \( r_{dg} \) the average outage duration of DG; and \( \lambda_{sw} \) the failure rate of manual switch; and \( s \) is the service restoration time from DG.

Service interruption rate and average interruption/restoration time are calculated for each load point by successive applications of series parallel reduction rules. This type of procedure is commonly employed in distribution systems reliability studies.

A different reliability modeling will be required if the DG capacity available is insufficient to fulfill the maximum demand of all the customers connected at a load point. In such situation the load point is partitioned in two portions one of it is supplied from main source as well as from DG as stand by and the other part is supplied from main source only. This reliability modeling aspect is shown in (see Fig. 2a). Partitioned load points are ‘i-A’ and ‘i-B’. In this case alternative supply is available for a fraction of customers connected at the load point. Figs. 2b and 2c shows reliability network for evaluating the indices at load point \( LP - iA \) and \( LP - iB \). Relations (1) and (2) along with series parallel reliability network reduction formulas are used to evaluate the basic indices at \( LP - iA \) or \( LP - iB \).

3. Problem formulation

Reliability indices may be improved for a radial distribution system by modifying failure rate and average repair time of each segment of the system. Addition of distribution generation (DG) may require additional cost per kWh to be paid to the private DG owners. In view of this the objective function is selected as follows

\[
J = \sum_{k=1}^{NC} \frac{r_k}{2} \sum_{k=1}^{NC} \frac{r_k}{2} + \text{ADCost(EENSO – EENS)D}
\]

In above \( \lambda_k \) is the failure rate of Kth section; \( r_k \) the average repair time of kth section; \( x_k \) and \( r_k \) are the cost coefficients; EENSO the
expected energy not supplied without DG; EENSD the expected energy not supplied with DG and ADCOST is the additional cost per kWh to be paid to DG owners.

The objective function constitutes of three parts i.e. (i) first part reflects cost of modifications of failure rates of each section. The quadratic form is based on the reliability growth model of Duane [17]. (ii) second part gives cost of modifications in average repair time of each section which is again based on a growth model of repair time. Larger is the repair time lesser is the cost of repair and vice versa, and (iii) third part represents additional cost to be paid to the owners of DG. This term in fact is the expected energy supplied (EENSO–EENSD) by DG multiplied by additional charge per kWh (ADCOSt). Thus the objective function provides a balance between additional cost incurred by the provision of DG and cost due to modifications in failure rates and repair times. It is to be noted that the operation of DG has been considered in standby mode and DG has been assumed to be privately owned. Hence power obtained from DG is costlier than that obtained from distribution substation. Other side one has to pay price for modifications in failure rates and repair times to achieve desired values of reliability indices as given by relations (7)–(10). Hence a compromise has to be achieved between failure rate and repair time modifications and additive cost of energy obtained from DG sources. This, in fact, is achieved by minimizing the objective function given by relation (3). This framed objective function includes the cost of ‘modifications’ and additional cost of energy purchased from DSO. Further note that as explained in Section 2, power cannot be tapped immediately from DG, but with a time delay of ‘s’ (service restoration time from DG) hours and this has been accommodated in modeling aspects. Another aspect which has been included that the demand may be fulfilled even partially in emergency conditions. This has been included in modeling aspects.

Expected energy not supplied (EENS) is calculated for all load points as follows [18]

$$\text{EENS} = \sum L_i U_{\text{SYS}_i}$$  \hspace{1cm} (4)

where $L_i$ is average load at $i$th load point; $U_{\text{SYS}_i}$ is average annual outage duration at $i$th load point.

EENS is calculated using relation (4) without DG is termed as EENSO and that calculated with DG is termed as EENSD. Expected energy not supplied for all load points is calculated using relation (4). In this relation $L_i$ is load at $i$th load point and $U_{\text{SYS}_i}$ is average annual outage duration at $i$th load point and is calculated using following relation for radial systems.

$$U_{\text{SYS}_i} = \sum_{k,S} \lambda_k r_k$$  \hspace{1cm} (4a)

$s$ denotes set of distribution segment in series up to $k^{th}$ load point; $\lambda_k$ is failure rate of $k$th segment and $r_k$ is average repair time for kth segment

Now to calculate $U_{\text{SYS}_i}$ accounting DG, the equivalent failure rate $\lambda_{eq}$ and equivalent repair time $r_{eq}$ are used as given by relations (1) and (2) in Section 2. Hence $U_{\text{SYS}_i}$ accounting DG is evaluated as follows

$$U_{\text{SYS}_i} = \lambda_{eq} \cdot r_{eq}$$  \hspace{1cm} (4b)

Whereas relation (4a) is used to evaluate $U_{\text{SYS}_i}$ and in turn used to evaluate EENSO (expected energy not supplied without DG).

Objective function (3) is minimized subject to following customer and energy based constraints [18]

(i) Bounds on the modification of failure rate and repair time of each section

$$\lambda_{k,\text{min}} \leq \lambda_k \leq \lambda_{k,\text{max}}$$

$$r_{k,\text{min}} \leq r_k \leq r_{k,\text{max}}$$  \hspace{1cm} (6)

(ii) Inequality constraint on system average interruption frequency index (SAIFI)

$$\text{SAIFI} \leq \text{SAIFI}_d$$  \hspace{1cm} (7)

System average frequency index (SAIFI) is defined as follows

$$\text{SAIFI} = \frac{\sum_{i} \lambda_{\text{SYS}_i} N_i}{\sum N_i}$$  \hspace{1cm} (8)

(iii) Inequality constraint on system average interruption duration index (SAIDI)

$$\text{SAIDI} \leq \text{SAIDI}_d$$  \hspace{1cm} (9)

SAIDI is defined as

$$\text{SAIDI} = \frac{\sum U_{\text{SYS}_i} N_i}{\sum N_i}$$  \hspace{1cm} (10)

(iv) Inequality constraint on customer average interruption duration index (CAIDI)

$$\text{CAIDI} \leq \text{CAIDI}_d$$  \hspace{1cm} (11)

CAIDI is evaluated as follows

$$\text{CAIDI} = \frac{\sum U_{\text{SYS}_i} \cdot N_i}{\sum \lambda_{\text{SYS}_i} N_i}$$  \hspace{1cm} (12)

(v) Constraint on average energy not supplied (AENS) per customer also known as average system curtailment index (ASCI)

$$\text{AENS} \leq \text{AENS}_d$$  \hspace{1cm} (13)

Where AENS is given as follows

$$\text{AENS} = \frac{\sum L_i U_{\text{SYS}_i}}{\sum N_i}$$  \hspace{1cm} (14)

In all above $N_i$ is the number of customers at $i^{th}$ load point; $L_i$ average load at $i^{th}$ load point; $\lambda_{\text{SYS}_i}$ is average failure rate at $i^{th}$ load point; $r_{\text{SYS}_i}$ is average interruption duration at $i^{th}$ load point; $U_{\text{SYS}_i}$ is average annual outage duration given as product of $\lambda_{\text{SYS}_i}$ and $r_{\text{SYS}_i}$.

$\text{SAIFI}_d, \text{CAIDI}_d, \text{SAIDI}_d$ and $\text{AENS}_d$ are threshold values of the SAIFI, CAIDI, SAIDI and AENS respectively. In Eqs. (5) and (6) the upper limits on failure rate ($\lambda$) and repair time ($r$) are considered as current (present) values as provided by any system data. Lower limits on failure rate ($\lambda_{\text{eq,min}}$) and repair time ($r_{\text{eq,min}}$) are the values which can be achieved in practice. In any case these values cannot be made zero. These lower limits are obtained in practice using failure
and repair data analysis along with the associated costs. The procedure is similar to that of reliability growth model [17]. The threshold values of reliability indices i.e. \(SAIFI, SAIDI, CAIDI, AENS\) are based on managerial/administrative decisions.

The formulated reliability optimization problem has been solved using robust population based optimization algorithms known as differential evolution (DE), particle swarm optimization (PSO) and one of its variant known as co-ordinated aggregation based particle swarm optimization (CAPSO). The formulated problem has been solved using these three optimization techniques so as to get confidence in the optimization results and applicability of these methods for solving such problem may be verified and comparison of the result may be made. The main idea of the paper is to obtain the optimum values of failure rates and repair times of each distribution segment subject to satisfaction of threshold values of reliability indices \(SAIFI, SAIDI, CAIDI, \) and \(AENS\) and incorporating the effects of distributed generation available in standby mode. The optimized values may be allocated as target values to preventive maintenance and corrective repair crew. Computational algorithms have been explained in next two sections.

### 4. Solution methodology using differential evolution (DE)

Storm and Price [19] developed a very simple population based stochastic function optimizer known as differential evolution (DE) technique. DE has been applied to solve various nature of engineering problems [20]. DE solves the optimization problem by sampling the objective function at multiple randomly chosen initial points. Bounds on decision variables define the region from which \(M\) vectors in the initial population are chosen. DE generates new solution points in \(D\) dimensional space that are perturbations of existing points. DE perturbs vectors with the scaled difference of two randomly selected population vectors. Further DE adds the scaled random vector difference to a third selected population vector (termed as base vector) to produce a mutated vector. A uniform crossover is employed to produce a trial vector from target vector and mutated vector. The DE based algorithm to solve the formulated problem is implemented in following steps.

**Step-1. Initialization:** Generate a population of size \(M\). Each vector consists of failure rates and repair times of each section of distribution system. The values of each component of vector are obtained by sampling uniformly between lower and upper limits as given by relations (5) and (6).

\[
\begin{align*}
X_i^0 &= [x_{i1}^0, x_{i2}^0, \ldots, x_{iNC}^0], \quad i = 1, \ldots, M \\
\bar{X}_i^0 &= [x_{i1}^0, x_{i2}^0, \ldots, x_{iNC}^0]
\end{align*}
\]

**Step-2. Evaluate \(r_{SYS}, \bar{r}_{SYS}\) and \(U_{SYS}\) at each load point with DG and without DG using series-parallel reduction formulas as explained in Section 2.**

**Step-3. Calculate reliability indices \(SAIFI, SAIDI, CAIDI, AENS\) at each load point with DG and without DG as defined in the relations 8,10,12,14, and 4 respectively.**

**Step-4. Evaluate objective functions \(J(X_i^0)\) for all vectors in the population as given by relation (3).**

**Step-5. Evaluate inequality constraints (7),(9),(11) and (13) for each vector of population. If a vector satisfies these constraints call it ‘F’ i.e. feasible, otherwise flag it as ‘NF’ i.e. not feasible. Identify a vector \(X_{i_{best}}^0\) i.e. a vector feasible and is best from objective function view point.**

**Step-6. Select a target vector, \(i = 1\).**

**Step-7. Select two random vectors \(X_p^k\) and \(X_q^k\) from the population, where \(p \neq q \neq i \neq best\).**

**Step-8. Generate a mutated vector using following relation**

\[
V_i^k = X_i^{best} + \sigma(X_p^k - X_q^k)
\]

\(\sigma\) is scale factor usually lies in the range \([0,1]\) and \(V_i^k\) is mutant vector.

**Step-9. If any component of mutant vector i.e. \(V_i^k\) violates the bounds on decision variables then bounce back mechanism [20] is applied. The bounce back method replaces violated element by the new element whose value lies between the base parameters (selected as \(X_{i_{best}}\)) value and bounds being violated as follows**

\[
v_{ij}^k = \begin{cases} x_{best,j} + \text{rand}(x_{j_{min}} - x_{best,j}), & \text{if } v_{ij}^k \leq x_{j_{min}} \\ x_{best,j} + \text{rand}(x_{j_{max}} - x_{best,j}), & \text{if } v_{ij}^k \geq x_{j_{max}} \end{cases}
\]

\(rand\) is random digit between \([0,1]\).

**Step-10. Trial vector \(X_i^{k+1}\) is obtained using uniform cross over as follows**

\[
t_{ij}^k = \begin{cases} v_{ij}^k, & \text{if } rand \leq C_r, \text{ or } j = j_{rand} \\ x_{ij}^k, & \text{otherwise} \end{cases}
\]

\(C_r\) is a crossover probability control parameter selected in the range \([0,1]\), \(C_r\) is user defined value which controls the number of decision variables which are copied from the mutant. \(rand\), is random digit in the range \([0,1]\).

**Step-11.** Performs reliability evaluation for trail vector \(X_i^{k+1}\) and obtain SFAI, SADI, CAIDI, and \(AENS\).

**Step-12. If all the inequality constraints i.e. 7,9,11, and 13 are satisfied then call the trial vector as ‘F’ otherwise flag it as ‘NF’.**

**Step-13. Obtain \(EENS\) and \(EENSD\) as given in relation (4).**

**Step-14. Evaluate objective function (3) for trial vector i.e. \(J(t_{ij}^{k+1})\) and \(J(t_{ij}^{k})\) is selected in new population under following conditions**

(i) \(J(t_{ij}^{k+1})\) is feasible and \(J(t_{ij}^{k+1}) \leq J(t_{ij}^{k})\).

(ii) \(J(t_{ij}^{k+1})\) is ‘F’ and \(X_{ij}^{k+1}\) is ‘NF’.

(iii) \(J(t_{ij}^{k+1})\) and \(X_{ij}^{k+1}\) both ‘NF’, but \(J(t_{ij}^{k+1})\) does not violate any constraint more than \(J(t_{ij}^{k})\). Otherwise \(X_{ij}^{k+1}\) is retained in new population.

**Step-15.** Select another target vector \(i = i + 1\).

**Step-16.** If \(i \leq M\), repeat from Step-7.

**Step-17.** If \(i \leq M\), repeat from Step-7.

**Step-18.** Increase generation count, \(k = k + 1\).

**Step-19.** If \(k < k_{max}\), repeat from step-6, otherwise stop. Where \(k_{max}\) is maximum number of generations specified.

It is stressed that DE algorithm may even be stopped before execution of maximum number of generation, if improvement in objective function is not observed in a pre-specified generations.

### 5. Solution methodology using particle swarm optimization

Particle swarm optimization was originally proposed by Kennedy and Eberhart [22]. The methodology is population based and applied to solve the formulated problem. The implementation of PSO for solving the reliability optimization problem is in following steps.

**Step-1.** Population of size \(M\) is generated where each particle contains failure rates and repair times of each section sampled uniformly within bounds governed by relation (5) and (6). Selected particles are assumed to be feasible i.e. these satisfy
inequality constraints 7.9,11, and 13. The generated particles are represented as

\[ X_{i}^{(l)} = [r_{1i}^{(l)}, r_{2i}^{(l)}, \ldots, r_{Ni}^{(l)}], r_{ni}^{(l)}, \ldots, r_{NC}^{(l)}], \quad i = 1, \ldots, M \]

Step-2. This step consist of initialization of velocities of each particle selected in step-1. The velocity of each element \( j' \) of particle \( i' \) is generated at random within the boundary given as follows

\[ \tilde{r}_{j_{i_{min}}}^{0} \leq r_{ji}^{0} \leq \tilde{r}_{j_{i_{max}}}^{0} \]

\[ \tilde{r}_{j_{i_{min}}}^{1} \leq r_{ji}^{1} \leq \tilde{r}_{j_{i_{max}}}^{1} \quad (18) \]

Step-3. The initial best position, \( P_{\text{best}_{i}}^{(0)} \), of particle \( i' \) is set as its initial position and initial best position of group, \( C_{\text{best}_{i}}^{(0)} \), is determined as the position of an individual with minimum value of objective function as obtained using relation (3).

Step-4. Set iteration count \( k = 1 \).

Step-5. Velocity of each individual (\( \rho_{i}^{(k)} \)) is modified using following relation

\[ \rho_{i}^{(k)} = \omega \cdot \rho_{i}^{(k-1)} + c_{1} \cdot \text{rand}_{1} \cdot (P_{\text{best}_{i}}^{(k-1)} - X_{i}^{(k-1)}) + c_{2} \cdot \text{rand}_{2} \cdot (C_{\text{best}_{i}}^{(k-1)} - X_{i}^{(k-1)}) \quad (19) \]

where \( P_{\text{best}_{i}}^{(k-1)} \) is the best previous position in the swarm; \( C_{\text{best}_{i}}^{(k-1)} \) is the global best position among all the participation in the swarm from objective function view point; \( \text{rand}_{1} \) and \( \text{rand}_{2} \) are random digit in the range \([0, 1]\); \( \omega \) is inertia weight and \( c_{1}, c_{2} \) are acceleration constants.

A large inertia weight \( \omega \) facilitates global exploration and a smaller inertia weight tends to facilitate local exploration to fine tune the current search area. Therefore the inertia weight \( \omega \) is an important parameter for convergence behavior of PSO. A suitable value of inertia weight usually provides balance between global and local exploration abilities and consequently results in a better optimum solution. In view of this it has been suggested that inertia weight \( \omega \) is varied as follows [23]

\[ \omega = \omega_{\text{max}} - \frac{\omega_{\text{max}} - \omega_{\text{min}}}{k_{\text{max}}} \cdot k \]

where \( k_{\text{max}} \) is the maximum number of iteration specified and \( k \) denotes current iteration. \( \omega_{\text{max}} \) and \( \omega_{\text{min}} \) denotes maximum and minimum values of inertia weight.

Step-6. Position of each particle, \( X_{i}^{(k-1)} \), is updated using following relation

\[ X_{i}^{(k)} = X_{i}^{(k-1)} + \rho_{i}^{(k)} \quad (20) \]

Step-7. If any component of updated particles violates the bounds given by 5.6 then it is set to limiting value.

Step-8. Perform reliability evaluation for each updated particle and evaluate SAII, SAII, CIAI, AEEN, EENS, and EENSD.

Step-9. Evaluate for each updated particle inequality constraints 7.9,11, and 13. If all these constraints are satisfied the updated particle is retained in updated population. If any of the constraints is not satisfied a fly-back mechanism [24] is adopted for handling the inequality constraints. This fly back mechanism the idea is to maintain a feasible population by flying back such updated particle to its previous position. Since the population is initialized in the feasible region flying back to its previous position will guarantee the solution to be feasible. Flying back to its previous position when an updated particle violates the constraints will allow a new search closer to boundary.

Step-10. Update \( P_{\text{best}_{i}}^{(k)} \) and \( C_{\text{best}_{i}}^{(k)} \) as follows

\[ p_{\text{best}_{i}}^{(k)} = \begin{cases} X_{i}^{(k)} | J(X_{i}^{(k)}) < J(P_{\text{best}_{i}}^{(k-1)}) \\ p_{\text{best}_{i}}^{(k-1)} \text{ otherwise} \end{cases} \quad (21) \]

\( G_{\text{best}_{i}}^{(k)} \) at this current iteration is set as the best value in terms of objective function among all \( P_{\text{best}_{i}}^{(k)} \).

Step-11. Increase iteration count \( k = k + 1 \)

If \( k \leq k_{\text{max}} \) repeat from Step-5, otherwise stop. PSO is a heuristic search procedure and as such no analytical convergence criterion exists, except that PSO is terminated when a pre-specified maximum number of iterations are executed. The maximum number of iteration may be specified based on computational experience [23]. Further the convergence may be observed by plotting a graph between \( J(G_{\text{best}_{i}}^{(k)}) \) and number of iterations.

6. Solution methodology using co-ordinated aggregation based particle swarm optimization (CAPSO)

CAPSO is an improved variant of conventional PSO [25]. Each particle moves considering only the positions of particles with better achievements than its own with the exception of the best particle which moves randomly. The coordinated aggregation may be considered as a type of active aggregation where particles are attracted only by places with the most food. Hence CAPSO differs from PSO described in previous section only in updating the velocity of each particle (Step-5 in previous section). Velocity of each particle except the best in swarm is updated using following relation

\[ \rho_{i}^{(k)} = \omega \cdot \rho_{i}^{(k-1)} + \sum_{j} r_{j} A_{j}^{(k-1)} \left[ X_{j}^{(k-1)} - X_{i}^{(k-1)} \right] \quad (22) \]

\( r_{j} \) denotes random digit between \([0, 1]\), \( A_{j}^{(k-1)} \) is known as achievement factor, which is the ratio of difference between achievement (objective function) of particle and better achievements by particle-\( j \) to the sum of all these differences. Expression for \( A_{j}^{(k-1)} \) is given as follows.

\[ A_{j}^{(k-1)} = \frac{J(X_{j}^{(k-1)}) - J(X_{i}^{(k-1)})}{\sum_{k=1}^{K} |J(X_{k}^{(k-1)}) - J(X_{i}^{(k-1)})|} \quad (23) \]

\( T \) represents the set of particles-\( j \) with better achievement in terms of objective function.

Relation (22) is used to update the velocities of all particles except the best particle in the swarm. The velocity of best particle in the swarm is updated using a random co-ordinator calculated between its position and the position of a randomly chosen particle in the swarm as follows.

\[ \rho_{p}^{(k)} = \omega \cdot \rho_{p}^{(k-1)} + r \left[ X_{p}^{(k-1)} - X_{p}^{(k-1)} \right] \quad (24) \]

\( X_{p}^{(k-1)} \) is randomly selected particle, \( X_{p}^{(k-1)} \) is the best particle in the swarm and \( r \) is random digit between \([0, 1]\). The updating of best particle of group behaves in a crazy way and helps the swarm to escape from local minima. Remaining steps are same as explained in previous section. CAPSO has better convergence characteristics and require only inertia weight parameter to start the updating procedure.

7. Results and discussions

The algorithm developed in this paper has been implemented on a sample radial distribution system as shown in Fig. 3 [11]. The system has seven load points in addition to a source sub-station and seven distributor segments. The system has been modified due to presence of DG at load points LP-3, LP-6 and LP-8. In the absence of loss of supply at these nodes continuity is maintained with the help of these distributed sources. Further it is assumed that DG-6 present at LP-6 is sufficient to feed the need of only 50% consumers connected at the load point and distributed generator DG-3.
and DG-8 connected at load points LP-3 and LP-8 are having sufficient capacities to meet demand of all the consumers connected at these load points. Table 1 gives current failure rates (\( \lambda_i \)), repair time (\( r^p_i \)) and bounds on these quantities for each section. Due to presence of distributed energy resources modifications on both sides of decision variables have been considered. Records of growth model of these variables allow to estimate bounds on these variables. Table 2 provides average loads and number of customers at each load point. Table 3 provides cost coefficient \( x_k \), \( \beta_k \) as required to evaluate objective function (J). The ADCOST is selected as Rs.1.5 per kW h.

Reliability modeling aspects are described in Section 2. \( \lambda_{SYS}, \tau_{SYS} \) and \( USYS \) for load points LP-2, LP-4, LP-5 and LP-7 (these do not have DG) are evaluated using following relation.

\[
\begin{align*}
&\lambda_{SYS,2} = \lambda_1 \\
&\tau_{SYS,2} = \tau_1 \\
&U_{SYS,2} = \lambda_1 \tau_1 \\
&\lambda_{SYS,4} = \lambda_1 + \lambda_2 + \lambda_3
\end{align*}
\]

Since distribution generations are present at LP-3, LP-6, and LP-8, the calculation of basic reliability indices will be different. The reliability network to evaluate basic indices for LP-3 is shown in Fig. 4 as per modeling aspects described in Section 2.

\[
\begin{align*}
&\lambda_{SYS,3} = (\lambda_1 + \lambda_2) \\
&\tau_{SYS,3} = \lambda_1 \tau_1 + \lambda_2 \tau_2 \\
&U_{SYS,3} = \lambda_1 \tau_1 + \lambda_2 \tau_2 + \lambda_3 \tau_3
\end{align*}
\]

In above \( \lambda_{SYS}, \tau_{SYS} \) and \( U_{SYS} \) with DG are obtained using relation (1) and (2) as follows.

\[
\begin{align*}
&\lambda_{SYS,3} = \lambda_{SYS,3} \lambda_{DG}(\tau_{DG} + \tau_{SW}) + \lambda_{SW} \\
&\tau_{SYS,3} = \lambda_{SYS,3} \lambda_{DG}(\tau_{DG} + \tau_{SW}) + \lambda_{SW} \tau_{SW,3} \lambda_{SYS,3}
\end{align*}
\]

Table 2

<table>
<thead>
<tr>
<th>Load point LP-K</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average load, ( L_k ), kW</td>
<td>1000</td>
<td>700</td>
<td>400</td>
<td>500</td>
<td>300</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>Number of customer</td>
<td>200</td>
<td>150</td>
<td>100</td>
<td>150</td>
<td>100</td>
<td>250</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_k ) (Rs.)</td>
<td>246</td>
<td>40</td>
<td>81</td>
<td>250</td>
<td>32</td>
<td>4.5</td>
<td>5.0</td>
</tr>
<tr>
<td>( \beta_k ) (Rs. ( \times 10^3 ))</td>
<td>100.00</td>
<td>72.90</td>
<td>72.00</td>
<td>200.00</td>
<td>180.00</td>
<td>38.40</td>
<td>93.60</td>
</tr>
</tbody>
</table>

**Fig. 3.** Radial distribution system with DG at selected load points.

**Table 1**

<table>
<thead>
<tr>
<th>Distribution section</th>
<th>( \lambda^p/1(\text{yr}) )</th>
<th>( \lambda_{min}/1(\text{yr}) )</th>
<th>( \lambda_{max}/1(\text{yr}) )</th>
<th>( r^p/\text{hrs} )</th>
<th>( t_{min}/\text{hrs} )</th>
<th>( t_{max}/\text{hrs} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.2</td>
<td>0.6</td>
<td>10</td>
<td>6</td>
<td>15.0</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.05</td>
<td>0.3</td>
<td>9</td>
<td>6</td>
<td>14.0</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.5</td>
<td>12</td>
<td>4</td>
<td>18.0</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.15</td>
<td>0.8</td>
<td>20</td>
<td>8</td>
<td>30.0</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>0.15</td>
<td>0.3</td>
<td>15</td>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>0.05</td>
<td>0.15</td>
<td>8</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>0.05</td>
<td>0.15</td>
<td>12</td>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>
only 50% consumers connected at LP-6. The reliability network 

Initial population generated for DE.

Failure rate and average down time for distributed generation.

\[ \lambda_{SW} = \lambda_1 + \lambda_2 + \lambda_6 + \lambda_8 \]

\[ r_{SW} = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_6 r_6 + \lambda_8 r_8}{\lambda_{SW}} \]

\[ \lambda_{SYS,8} = \lambda_{SW} \cdot \lambda_{DG} + \lambda_{DG} (r_{SW} + r_{DG}) + \lambda_{SW} \]

\[ r_{SYS,8} = \frac{\lambda_{SW} \cdot \lambda_{DG} (r_{SW} + r_{DG}) + \lambda_{SW} \cdot S_8}{\lambda_{SYS,8}} \]

\[ \text{DG} \text{ Capacity available is sufficient to satisfy the requirement of only 50\% consumers connected at LP-6. The reliability network for LP-6 is shown in Fig. 4. LP-6 is partitioned in two separate nodes i.e. 6A and 6B. 50\% consumer may be supplied by either distribution network or DG and remaining are supplied by DG-6 in case of loss of supply from source node. The basic reliability indices are evaluated for LP-6A using following formula.} \]

\[ \lambda_{SYS,6A} = \lambda_1 + \lambda_4 + \lambda_5 \]

\[ U_{SYS,6A} = \lambda_1 r_1 + \lambda_4 r_4 + \lambda_5 r_5 \]

\[ r_{SYS,6A} = \frac{U_{SYS,6A}}{\lambda_{SYS,6A}} \]

Basic reliability indices at LP-6B is evaluated using following relations as given by relations (1) and (2).

Table 4

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>DG No.</th>
<th>Failure Rate ( \lambda_{DG} ) (1/yr)</th>
<th>Down time ( r_d ) (h)</th>
<th>Failure rate of manual switch ( \lambda_{sw} ) (1/yr)</th>
<th>Service restoration time ( s ) (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DG-3</td>
<td>1.50</td>
<td>15.78</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>DG-6</td>
<td>1.75</td>
<td>18.68</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>DG-8</td>
<td>2.00</td>
<td>20.61</td>
<td>0.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

\[ \lambda_{DG} \text{ and } r_{DG} \text{ are failure rate and average interruption duration of DG-8.} \]

\[ \lambda_{SYS,8} \text{ and } S_8 \text{ are failure rate of manual switch and service restoration time for DG-8.} \]

Table 5

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
<th>#9</th>
<th>#10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_1 )</td>
<td>0.2585</td>
<td>0.3914</td>
<td>0.3230</td>
<td>0.4674</td>
<td>0.4799</td>
<td>0.3470</td>
<td>0.2565</td>
<td>0.3955</td>
<td>0.3186</td>
<td>0.2985</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>0.0636</td>
<td>0.1299</td>
<td>0.2093</td>
<td>0.1163</td>
<td>0.1278</td>
<td>0.1009</td>
<td>0.0955</td>
<td>0.2051</td>
<td>0.0523</td>
<td>0.0844</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>0.2208</td>
<td>0.4862</td>
<td>0.4127</td>
<td>0.4514</td>
<td>0.1356</td>
<td>0.2651</td>
<td>0.3874</td>
<td>0.1332</td>
<td>0.1849</td>
<td>0.2336</td>
</tr>
<tr>
<td>( \lambda_4 )</td>
<td>0.2217</td>
<td>0.5069</td>
<td>0.7234</td>
<td>0.5934</td>
<td>0.3358</td>
<td>0.6560</td>
<td>0.3548</td>
<td>0.4777</td>
<td>0.4668</td>
<td>0.4535</td>
</tr>
<tr>
<td>( \lambda_5 )</td>
<td>0.2749</td>
<td>0.2705</td>
<td>0.2323</td>
<td>0.2777</td>
<td>0.2767</td>
<td>0.1829</td>
<td>0.1969</td>
<td>0.2419</td>
<td>0.2273</td>
<td>0.2082</td>
</tr>
<tr>
<td>( \lambda_7 )</td>
<td>0.0767</td>
<td>0.0506</td>
<td>0.0939</td>
<td>0.1408</td>
<td>0.1374</td>
<td>0.1409</td>
<td>0.1448</td>
<td>0.0984</td>
<td>0.0501</td>
<td>0.0620</td>
</tr>
<tr>
<td>( r_4 )</td>
<td>8.8760</td>
<td>27.5932</td>
<td>8.5542</td>
<td>13.7090</td>
<td>29.5521</td>
<td>27.6815</td>
<td>23.9893</td>
<td>11.3934</td>
<td>11.4329</td>
<td>24.9299</td>
</tr>
</tbody>
</table>

Objective function, \( f \times 10^4 \)

<table>
<thead>
<tr>
<th>F/NF</th>
<th>F</th>
<th>NF</th>
<th>F</th>
<th>NF</th>
<th>F</th>
<th>NF</th>
<th>F</th>
<th>NF</th>
<th>F</th>
<th>NF</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2411</td>
<td>1.0406</td>
<td>0.9647</td>
<td>0.8031</td>
<td>1.2908</td>
<td>1.1411</td>
<td>1.3185</td>
<td>1.3219</td>
<td>2.0575</td>
<td>1.8889</td>
<td></td>
</tr>
</tbody>
</table>

\( NF \) denotes not feasible solution.

\( F \) denotes feasible solution.
restoration time of DG-6.

SAIFI, SAIDI, CAIDI and AENS as given by relations 8, 10, 12, and 14 respectively. Similarly EENSO and EENSd are also evaluated using relations 4, 4a, 4b.

Table 4 gives failure rate, average down time for distributed generations present at load points 3, 6 and 8. It also provides failure rate ($\lambda_{sw}$) and service restoration time ($s_d$) for manual switch of these distributed generations. In evaluating EENSO (Without DG) the DG at location 3, 6, and 8 load points are not considered. The problem as formulated in Section 3 has been solved using DE, PSO and CAPSO based algorithm as explained in Sections 4–6. Table 5 gives initial population generated for DE. Table 6 presents the initial population generated and used in PSO and CAPSO algorithms. These populations were generated with an inbuilt random number generator program. Table 7 gives the final control parameter selected for DE, PSO and CAPSO (see Fig. 6).

Table 8 gives optimum failure rate and repair time of each section as obtained by these three methods, along with cost function. It is observed that results obtained by these three methods are in close agreement. Further in DE based optimization all the members of population become feasible and cluster around the optimum solution at the end of convergence. This has been an important characteristics of the solutions obtained by DE and has been pointed out in Ref. [20]. This has been also observed by us in our computational procedure at the end. This has been put in Table 9. Table 9 gives CPU time required to solve the optimization problem by three methods with various population sizes. It was observed that PSO converges with even five population size whereas DE does not converge with five population size. Solution with DE converges with ten population size. But as population size is increased CPU time with PSO increases from 78 ms to 281 ms with population size 5

Table 8

Optimized values of failure rates and repair times as obtained by DE, PSO and CAPSO.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Magnitudes as obtained by DE</th>
<th>Magnitudes as obtained by PSO</th>
<th>Magnitudes as obtained by CAPSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$ (yr)</td>
<td>0.2331</td>
<td>0.2471</td>
<td>0.2245</td>
</tr>
<tr>
<td>$\lambda_2$ (yr)</td>
<td>0.1789</td>
<td>0.1357</td>
<td>0.1743</td>
</tr>
<tr>
<td>$\lambda_3$ (yr)</td>
<td>0.3104</td>
<td>0.2932</td>
<td>0.3449</td>
</tr>
<tr>
<td>$\lambda_4$ (yr)</td>
<td>0.3430</td>
<td>0.3685</td>
<td>0.3731</td>
</tr>
<tr>
<td>$\lambda_5$ (yr)</td>
<td>0.2845</td>
<td>0.2998</td>
<td>0.2719</td>
</tr>
<tr>
<td>$\lambda_6$ (yr)</td>
<td>0.0920</td>
<td>0.0952</td>
<td>0.0901</td>
</tr>
<tr>
<td>$\lambda_7$ (yr)</td>
<td>0.1483</td>
<td>0.1487</td>
<td>0.1239</td>
</tr>
<tr>
<td>$r_1$ (h)</td>
<td>6.0033</td>
<td>6.0023</td>
<td>6.4414</td>
</tr>
<tr>
<td>$r_2$ (h)</td>
<td>6.0767</td>
<td>6.0012</td>
<td>6.7191</td>
</tr>
<tr>
<td>$r_3$ (h)</td>
<td>6.1595</td>
<td>13.2311</td>
<td>8.6474</td>
</tr>
<tr>
<td>$r_4$ (h)</td>
<td>8.0100</td>
<td>8.0029</td>
<td>8.1497</td>
</tr>
<tr>
<td>$r_5$ (h)</td>
<td>7.0643</td>
<td>7.0685</td>
<td>7.2750</td>
</tr>
<tr>
<td>$r_6$ (h)</td>
<td>9.2649</td>
<td>11.3771</td>
<td>6.1672</td>
</tr>
<tr>
<td>$r_7$ (h)</td>
<td>8.3064</td>
<td>6.0047</td>
<td>6.7913</td>
</tr>
<tr>
<td>Objective function</td>
<td>1.1701</td>
<td>1.1836</td>
<td>1.1841</td>
</tr>
</tbody>
</table>

$J_{SYS_6} = \frac{r_{SYS_6} \cdot \lambda_{dg} + \lambda_{sw}}{\lambda_{SYS_6}}$ + $J_{SYS_6}$

$J_{SYS_6} = \frac{r_{SYS_6} \cdot \lambda_{dg} + \lambda_{sw}}{\lambda_{SYS_6}}$ + $J_{SYS_6}$ + $J_{g_6}$

In above $\lambda_{dg}, \lambda_{sw}$ are failure rate and average down time of DG-6. $J_{g_6}$ is failure rate of manual switch of DG-6. $s_g$ denotes average service restoration time of DG-6.

The basic indices evaluated as above are used to evaluate SAIFI, SAIDI, CAIDI and AENS as given by relation 8, 10, 12, and 14 respectively. Similarly EENSO and EENSd are also evaluated using relations 4, 4a, 4b.

Table 9

CPU Time for convergence required for DE, PSO, and CAPSO Techniques on Intel core2duo processor, 2.9 GHz.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Techniques</th>
<th>Population size</th>
<th>CPU time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DE</td>
<td>10</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>PSO</td>
<td>05</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>281</td>
</tr>
<tr>
<td>3</td>
<td>CAPSO</td>
<td>05</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>250</td>
</tr>
</tbody>
</table>

NF denotes not feasible solution.
F denotes feasible solution.
and 20 respectively. The same is observed with DE and CAPSO. For population size 20 the CPU time required by DE is less than that required by PSO but equals to that required by CAPSO. In all the three methods CPU time increases as population size increases. Table 10 gives reliability indices as obtained using current value of decision variables as well as those obtained using optimized variables by three methods. Figs. 7–9 shows convergence curves as obtained by three methods for different population sizes.

### 8. Conclusions

This paper has presented a methodology for reliability design of a radial distribution system in the presence of distributed generation. An optimization problem has been formulated which obtains optimum failure rates and repair times of each section subject to satisfaction of inequality constraints based on energy and customer based reliability indices. Operating philosophy of DG has been assumed in standby mode and cost of purchased energy is more than that supplied from distribution substation. Formulated problem has been solved using two most robust and popular population based algorithm, i.e. DE and PSO. The solution has also been obtained using CAPSO, one of the variant of PSO. The results obtained have been compared. PSO requires less CPU time (population size 5) even with less population size than used in DE (population size 10). It was also observed that CPU time required for convergence increases with population size for all the three techniques. It was also observed that for population size twenty CPU time required by DE is less than that required by PSO.

### References